Wage Signaling, Salary History Bans, and Equality

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We develop a model of employee mobility and wage formation suitable for analyzing labor market interventions that target wage inequality. By augmenting the standard asymmetric learning model with costly effort, such that wages are necessary for production, wages become fully informative of employee ability rather than merely products of public signals and employer competition for workers. Strategies designed to mask employee quality, commonly seen in standard models, are untenable with salary disclosure. The equilibrium with salary disclosure has symmetric learning, optimal employee assignment, and efficient production. When salary disclosure is prohibited, as is the current policy in many jurisdictions, the equilibrium reverts to one of asymmetric learning, with strategic under-assignment due to manipulation of noisy signals. Equality mandates, such as published wage ranges and lockstep compensation, prevent firms from fully utilizing more skilled employees, resulting in within-title underutilization, a novel source of inefficiency.

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I. Introduction

Policy makers in the United States and abroad are increasingly enacting labor market interventions designed to combat historical inequities in the distribution of wages across employees (Cullen 2023). One prominent intervention is the salary history ban, a recent regulatory reform in important labor markets in the United States. Bans seek to suppress the information content in wages by prohibiting firms from asking prospective employees about their prior salaries. They are intended to have beneficial effects for workers, particularly women and minorities, by reducing "anchoring," whereby unfairly low wages follow employees throughout their careers.[2]

We refer to another broad category of interventions as "equality mandates;" these seek to reduce the variation in wages across employees through various means. Pay transparency laws are a common example; some versions require employers to post a range of wages for any job they advertise. These postings cabin potential wages to limit the ability of employers to discriminate against prospective employees, and provide transparency to existing employees about the prevailing market wage for their role.[3] Similarly, some jurisdictions require employers to disclose internal pay disparities, under the theory that public disclosure of a pay gap will force companies to minimize unjustified wage disparities across gender or other categories.[4] Giving bite to such policies are recent court decisions focused on gender and racial pay equality, which require employers to demonstrate that pay disparities are based on acceptable factors (i.e., not sex or race) and which disallow past pay as an acceptable factor.[5] Equity advocacy groups have

[2] While one might expect informational unraveling, which leads to complete voluntary disclosure, it is the stated purpose of these laws to prevent use of such information. Accordingly, some of these laws, such as those of California and Massachusetts, render voluntarily disclosed salary histories problematic. Voluntary disclosure is also potentially subject to later claims of illegal "prompting." As a result, certain large employers, such as Google, Facebook, and Amazon, have stated that they will no longer utilize salary histories.

[3] Such laws are in effect in Colorado and New York, amongst other jurisdictions.

[4] The United Kingdom is one such jurisdiction; it requires employers with more than 250 employees to disclose their gender pay gap annually.

[5] A recent 9th Circuit opinion interpreted the Equal Pay Act to required employers to prove that pay differentials are based upon valid factors, such as productivity, as opposed to sex, and disallowed the use of salary histories as such a valid factor; the minority dissent noted that this rule would "demand a lockstep pay system." Rizo v. Yovino, Case No. 16-15372, 9th Circuit, February 27, 2020.

attacked pay disparities, and recommend practices such as lockstep compensation, limiting managerial discretion in setting pay, and standardizing compensation.[6]

These reforms, despite their good intentions, raise serious questions. Why might employers rely on salary histories in the first place? Will banning salary histories actually lead to greater pay equality? More broadly, how will forced pay equalization affect productivity? What are the distributional consequences, and how are returns to human capital impacted?

In this article, we develop a theoretical framework to analyze the effects of such interventions on the labor market, which existing models cannot do. The model presented here develops a link between employee skill, pay, and productivity. As we show, salary histories are informative, and while banning them leads to less inequality, it does so at the cost of adverse selection and lower productivity. Similarly, equality mandates lead to lower productivity and lower returns to skill.

To generate a link between wages, productivity, and skill, our framework makes two adjustments to the standard asymmetric learning model. First, we add disutility of effort, with disutility increasing along the intensive margin of employee production. This simple but fundamental import from contract theory implies that higher wages are required for increased production, because higher skilled employees must be paid sufficiently to motivate harder work. Second, we assume that task assignments are continuous, rather than discrete as is typical in the literature. Outside firms cannot observe the precise assignments, but they can observe a noisy signal of assignment, akin to the "promotion" threshold that is standard in the literature.

We show that with these adjustments, wages are perfectly informative of employee skill. When wages can be disclosed, the disutility of effort ensures that wages fully reveal employee ability. Unlike in the standard literature, strategic underpromotion or other masking strategies are untenable. The difference between our conclusion and the standard result is driven by the fact that employees maximize utility net of effort, rather than just their wage. Conceptually, an incumbent employer cannot allow a negative utility gradient to exist with respect to skill, as highest-skill employees would be easiest to poach. An incumbent firm that paid a pooling wage to mask the skill of certain employees loses its highest skilled (but lowest utility) employees to

[6] These recommendations come from the California Governor's Commission on the Status of Women and Girls. See, What Can I do to Promote a Culture of Pay Equity?, https://women.ca.gov/californiapayequity/employers-resources/what-can-i-do-to-promote-a-culture-of-pay-equity/.

competing firms that match the pooled wage but pair it with a job assignment that is utility-increasing only for highly skilled employees, who will earn the same wage but exert less effort. Lower skilled employees reject the offer because it pays the same wage but requires more effort. Outside firms would earn profits from the highly skilled employees they poached, and the incumbent firms would be left with overpaid lower skilled employees. As a result, a viable equilibrium requires that incumbent firms pay their skilled employees higher wages. Competing firms induce employee quality from the wage, and they bid up wages to marginal product.[7] The result is equivalent to a competitive equilibrium with symmetric information: employees earn their full (outside) marginal product every period, firms make no profits (excepting those linked to firm-specific human capital), wages start low and rise for skilled employees over time, and the full productive capacity of the economy is realized each period. This occurs despite the absence of any extrinsic public signal. Since wages are fully informative, noisy signals of employee assignment are irrelevant. Our analysis provides a theoretical grounding for the empirical observation that, in the status quo ante, many employers asked about prior wages when making lateral hires. Hall & Krueger (2012) report that "about half" of workers reported that their employers learned their salary histories before extending job offers, and Barach & Horton (2017) find that for workers whose employers learned their histories, such learning occurred prior to extending the job offer in more than 80% of cases.[8]

Having demonstrated the informativeness of wage histories, we then show how salary history bans jam the wage signal and lead to equilibria with inefficient utilization of employees. A ban directly prohibits outside firms from utilizing the wage signal, which reduces the outside option available to skilled employees, allowing incumbent employers to pay only the minimum wages necessary to induce effort. This reduces the return to skill, but re-introduces reliance on noisy signals of employee assignment, and the associated inefficiencies linked to manipulation of

[7] Disutility of effort presents another challenge to pooling equilibria: if pooling were viable, lower skilled employees would earn higher utility than higher skilled employees. This would create disincentives for high-ability employees to reveal their ability in the first place, which would require them to accept utility-decreasing promotions. Although we do not rely on such a revelation mechanic in our model, we do think it illustrates another reason why costly effort renders masking and wage pooling strategies untenable.
[8] This is, of course, not the only plausible theory for requesting salary history data. Bessen, Meng & Denk (2020) posit that employers request salary history because it reveals heterogeneous reservation wages among otherwise identical workers.

these signals; incumbent firms strategically underpromote some skilled employees, in keeping with the standard asymmetric learning models.

We make a number of empirical predictions regarding the implications of bans based on this analysis. First, we expect that the evidence for asymmetric learning will be stronger in jurisdictions that enact salary history bans. Our assumptions regarding costly effort and continuous tasks are plausible for any jobs where the intensive margin of employees varies significantly (e.g., deal volume of an investment banker), as opposed to occupations where the work is largely commoditized (e.g., instructors teaching standardized courseloads). We predict less evidence of asymmetric learning in jobs with significant within-title wage variation when wages can be disclosed, and greater evidence after the imposition of a ban. That said, we expect bans to reduce ex-post income inequality, due to the reduction of the outside option and associated reduction in the return to skill earned by more skilled employees. Again, this effect will be more severe in roles where there is significant intra-title variation in tasks and in pay. Bans will also lead to increased wages for new entrants into the labor market. Under a ban, promotions are inefficiently low, but within a given title employees are utilized up to their capacity (or the capacity inherent in their role). Without the wage signal to drive up their outside option, employees earn wages that are below their marginal product. This implies that firms make positive expected lifetime profits from employees, which they distribute in the form of higher starting wages as they compete for new hires.

We then examine a broad range of policies designed to induce wage equality. We find that reductions in the variance of pay across employees lead to underutilization of employees, but through a subtler (and new) channel. Because tasks are continuous, but the "promotion" threshold is discrete, employees with a range of ability can have the same title. Equality mandates require firms to base pay on observable factors such as title, rather than unobservable factors like ability. This forces firms to pool employees: workers with different ability levels earn the same wage. However, since effort is costly, firms cannot fully utilize the more-skilled employees within a given job title because causing such employees to exert more effort would create a negative utility gradient in skill. Outside firms could exploit this to poach the more-skilled employees. Conceptually, by constraining wages within titles, an equality mandate also constrains the maximum feasible assignment of each title. The result is within-title underutilization of employees, a novel source of inefficiency, and a corresponding reduction in

the average wage and average productivity of employees. This conclusion accords with recent empirical analysis of intra-firm pay transparency and related wage-equality initiatives (Cullen 2023).

Like a salary history ban, equality mandates reduce wage inequality at the cost of lower aggregate welfare. Skilled employees are worse off because they cannot be utilized, and paid, according to their ability. However, employees are paid wages (approximately) equal to their realized, as opposed to potential, outside marginal product, because the wage signal is informative to outside firms. This limits the lifetime profits and the associated increase in initial wages paid to new entrants to the labor market. Although promotion decisions are generally inefficient under an equality mandate, both under-assignment and over-assignment are possible. The latter is driven by the "wedge" between feasible assignments at each title. When the maximum feasible assignment for employees who are not promoted is low, it may be worthwhile to over-assign employees with skill close to, but below, the public promotion threshold, if the losses from over-assignment are below the gains from greater utilization of their skill.

The rest of the article proceeds as follows. In Section II, we relate our model to the prior literature on employer learning and emerging research on the effect of salary history bans and other equality initiatives. In Section III, we develop a formal model of wages and mobility with asymmetric learning and costly effort. This model draws on the standard models of asymmetric employer learning (Waldman 1984, Greenwald 1986, DeVaro and Waldman 2012), in which incumbent employers observe employee quality but competitor firms receive only limited signals. The primary additions are the disutility of employee effort, with greater effort required for higher production, and a continuous distribution of tasks. In Section IV, we solve for the equilibria assuming wages can be disclosed. The main result is that the disutility of effort associated with increased production leads to revealing wages: with salary disclosure, the unique equilibrium is equivalent to symmetric learning, and wages equal marginal product. In Section V, we derive the implications of suppressing the wage signal; we separately consider salary history bans and equality mandates. Each introduces inefficiencies; the salary history ban leads to an equilibrium of asymmetric learning with noisy public signals, whereas the equality mandate leads to constraints on the feasible assignments within each title. Section VI discusses our results and paths for future work. Section VII concludes.

II. Relation to the prior literature

A. Theoretical models of employer learning

This article builds on the significant theoretical labor economics literature on asymmetric learning. [9] This literature seeks to explain the role of adverse selection in employee job mobility and the potential negative effects of competitive hiring pressure, which create incentives to deter poaching of employees (Greenwald 1986, Waldman 1984). In the standard approach, public signals are noisy and endogenous, and inefficiencies result from manipulation of the endogenous signal.[10] A typical assumption is that outside firms can observe job assignments (Waldman, 1984) or promotions (DeVaro and Waldman, 2012). In equilibrium, employers underpromote their high skill or high productivity workers to deter poaching by competitor firms (Waldman & Zax (2016) demonstrate the robustness of this result to a range of modelling assumptions). Wages do not generally reflect an employee's marginal product, because competitor firms fail to learn about employee ability. This results in high wages for new entrants to the labor market, as firms compete away future profits they expect to earn from new hires.

The existing models provide limited insight into the effects of labor market interventions designed to regulate or impinge on wage formation, or that seek to suppress the information contained in wages. In the standard approach, wage disclosure (or lack thereof) is irrelevant: wages reflect public signals and are not themselves incrementally informative (Waldman 1984). In more recent studies, wages are assumed to have signaling value, and banning disclosure can affect the distribution of bargaining power between firms and employees. For example, in Golan (2009) wages serve as a signal of productivity; differential wages arise from an extrinsic bargaining game over surplus between employer and worker. Pinkston (2009) presents an asymmetric learning model with wage disclosure, where wages are a function of exogenous signals and labor market competition. However, suppressing the information content in wages does not lead to efficiency losses, because productivity is only tenuously linked to wage

[9] If learning was symmetric, then none of these laws would have any effect on wages; therefore, the asymmetric learning models are the natural place to start an analysis.

[10] In some studies, the signals are assumed to be exogenous but noisy. For example, Schonberg (2007) considers noisy, exogenously-generated, public signals; greater noise generates more adverse selection in worker mobility.

formation: the productivity of any individual employee is unaffected by the wage.[11] Without competition, employers in these models would need not pay differential wages. Further, in these models, policy goals designed to achieve wage equality could be achieved by government fiat – for instance, mandating a set wage for all worker demographics, or by outlawing competition for workers – without loss of efficiency. In fact, some such policies create efficiency gains in these models. For example, in Waldman (1984), the government could mandate either a wage of zero for all employees or forbid competition for employees. This improves efficiency: Waldman's firms otherwise mismatch employees to jobs in order to limit poaching.[12] Overall, current models cannot address the recent wave of reforms that attempt to modify processes of wage formation and competition in labor markets, such as salary history bans and various wage equality initiatives.

To that end, we put forth a parsimonious model in which wages are an input into production, such that wages then serve as signals of quality. As in Waldman (1984) and much of the literature that follows, in our model firms divide their employees' time between two tasks: an unskilled task and a skilled task that is more productive for high ability employees. As is also standard in these models, incumbent firms learn about their own employees' capabilities at the skilled task over time, whereas outside firms do not. Our addition is that the skilled task requires costly effort, increasing with the amount of the task assigned to the employee, and we model the wage as publicly observable.

To be clear, ours is not the first study to consider costly effort in the context of asymmetric learning. Gibbs (1995) analyzes a model in which production is determined by the product of effort and ability. More recent articles assume that production is a function of the sum of effort and ability, including Ghosh and Waldman (2010) and Ekinci, Kauhanen, and Waldman (2018), both of which analyze how optimal contracting relates to learning, effort, and public signals. However, these studies either implicitly (e.g., Gibbs, 1995) or explicitly (e.g., Ekinci, Kauhanen, and Waldman, 2018) assume that wages cannot be disclosed. For example, in the equilibria computed in Ekinci, Kauhanen, and Waldman (2018), the first period compensation

[11] More specifically, once the reservation wage is met, employees will produce at any feasible level regardless of the wage. In contrast, the contract theory literature considers settings in which compensation schemes are necessary to promote optimal employee production. Bolton and Dewatripont (2004) provide a survey of such models.

[12] Eliminating all returns to skill could result in other inefficiencies, such as reducing the incentive for workers to invest in their human capital. Banning wage disclosure has a similar effect in our model.

(wage plus bonus) would be fully informative of employee ability, could it be disclosed to outside firms. Ours is the first article to explicitly study how costly effort affects the information content of wages, how this interacts with wage formation and learning, and how various interventions into wage formation and disclosure affect job assignments, wages, and efficiency.

B. Early empirical work on salary history bans equality mandates

Our model also helps explain recent empirical findings regarding salary history bans, requirements for wage ranges in job listings, and pay transparency. Where salary history bans are enacted, our model predicts that the evidence for asymmetric learning will become relatively stronger in such jurisdictions, particularly for jobs in which higher wages are required to motivate greater productive effort. We expect that pooling and masking strategies will become relatively more prevalent in those jurisdictions, and that mobility will increase among lower wage workers. Low wage, low skill workers will see a rise in pay, while high pay, high skill workers will see declines; overall, productivity falls. Equality mandates have similar effects, with low skill, low pay workers gaining at the expense of higher skill workers and overall productivity.

These predictions are broadly consistent with empirical work to date. Initial studies of salary history bans have largely explored demographic cleavages, such as the impact on the gender and minority wage gaps. Agan, Cowgill & Lee (2021) find that salary history bans lead to less inequality in salary offers, with lower salary offers overall. Bessen, Meng & Denk (2020) find that salary history bans generally increased wages, with disproportionate benefits accruing to white females, non-white males, and white male job-switchers. Davis, Ouimet & Wang (2021) find indications that salary history bans made employers more cautious in their offers to new hires, offering lower wages as a result. Hansen & McNichols (2020) detect positive effects concentrated on women over 35 or with children. Mask (2020) examines "scarred" workers (those who employment history is hampered by a recession) and observes that they enjoy increased mobility due to salary history bans. Sinha (2019) provides evidence that salary history bans benefited females and hurt males, while an update (Sinha (2022)) finds effects only on white females (positive). Khanna (2020) provides experimental evidence that the ability to

volunteer salary history data frustrates their intended purpose.[13] Data from Sran, Vetter & Walsh (2020) show new hiring declining after the implementation of a salary history ban. Cullen & Pakzad-Hurson (2021) find that that salary transparency rules (specifically, allowing workers to share salary information with one another) lead to lower overall salaries, due to employer incentives to reduce salary inflation. Cullen (2023), in a survey of recent literature, reports similar results, noting that "'horizontal' pay transparency policies that reveal pay gaps between co-workers at the same firm… lower worker bargaining power and wages," while noting that providing information to workers about job opportunities across the landscape of firms appears to raise wages.

Overall, these results suggest increased wage equality but at the cost of efficiency losses, reflected by lower overall wages (though we note that there is not uniformity in these findings). Further, the measured effects appear to be largely distributional, with transfers from higher wage to lower wage workers (such as from white to non-white and male to female).

III. The model: wage formation with costly effort

Our approach is based on standard asymmetric learning models, with two major modifications. Firstly, and most importantly, we assume that that effort is costly, and that increased production requires greater effort. This differs from other models with costly effort, in which effort is additive to production but not required for it.[14] We assume that firms can observe the effort exerted by their employees.[15] This allows us to abstract from the monitoring or contracting necessary to incentivize effort, to focus on the information content in wages.

As in the standard models, we assume that firms derive production from two tasks, one of which is potentially higher value. The second major departure from the standard approach is that we assume that task assignments are continuous: employees split their time between the different

[13] As we note in a separate paper, there are reasons to believe that such volunteering is limited. Several salary history ban laws appear to limit the usage of any salary history information, and some prominent employers have stated that they will not utilize salary history information at all. Meli & Spindler (2019). Davis, Ouimet & Wang (2021) also provide evidence that salary history bans may change employer norms and practices, such that salary histories may not be utilized even when technically allowed (as it often is in the public sphere).

[14] Our assumption is similar in spirit to the assumption in Gibbs (1995) in which production is the product of effort and ability.

[15] Our analysis would be qualitatively identical if effort was only imperfectly observable, with the only adjustment being that the wage necessary to induce effort would need to be adjusted higher to reflect the probability of being caught shirking (an efficiency wage).

tasks, rather than perform only one or the other task. We assume that the precise assignments cannot be credibly disclosed to outside firms. This is motivated by the continuous nature of the assignments, as outside firms would struggle to verify how exactly an employee's time was allocated across tasks. However, we do assume that there is a "promotion" threshold that is observable by outside firms, akin to the promotions considered in the standard models. If an employee's assignment to the higher value task exceeds this threshold, outside firms recognize her as having been promoted. For example, we can think of the higher value task as management of other employees. An employee assigned a small amount of management might have the same title as some employees on her team (e.g., "Associate" at a law firm, or "Vice President" at an investment bank). This employee would not be able to credibly disclose her precise management responsibilities to outside firms. However, management responsibilities above a certain threshold come with a different title (e.g., "Partner" or "Managing Director") that can be disclosed.

A. The players

We assume there exists a continuum of profit maximizing firms, which have the same constant returns to scale production function that spans two tasks, both of which require only labor as an input. We assume a mass of employees have a unit of time to spend at work each period, which they divide between the two tasks.

One task is "unskilled." It requires no effort and generates the same expected production (per unit of time) for every employee. Without loss of generality, we normalize the expected productivity of the unskilled task to 0.[16] The second task is "skilled," which requires (observable) effort e per unit of time to be successfully completed (i.e., we assume a linear cost of effort). Each employee i is endowed with a capacity for the skilled task of $\alpha_i \in [0,1]$. The probability distribution function of skill in the population is given by $f(\alpha)$, with expected value $E[\alpha]$. This distribution is known to the players. The per-unit of time expected production at the skilled task is $y > e$, so long as the employee spends no more than α_i of her total time at work that period on the skilled task. Any allocation beyond α_i generates negative expected production. We assume that employees develop firm-specific human capital that increases production if the

[16] The necessary assumption is that both the effort level and the productivity (when performed up to the capability of an employee) of the skilled task are above those of the unskilled task. Then we normalize both effort and productivity accordingly.

employee stays with the same firm for more than one period. Production per unit of time in the unskilled task rises by $s_0 > 0$, and in the skilled task by a factor $s_1 > 0$. We assume that $y * s_1 > s_0$; firm specific human capital does not change the ordinal relationship between the productivity of the two tasks.

Formally, an employee *i* endowed with skill α_i, employed by firm *j*, and assigned to spend a fraction $\beta_{i,t}$ of her work hours in period *t* at the skilled task (and the rest at the unskilled task), generates expected production $E[Y_{i,j,t}]$ of:

$$E[Y_{i,j,\mathrm{t}}] = \left(1 - \beta_{i,t}\right) * s_0 * I_{i,j,t-1} + \min\left(\beta_{i,t}, \alpha_i\right) * y * \left(1 + s_1 * I_{i,j,t-1}\right) + \max\left(\beta_{i,t} - \alpha_i, 0\right) * L \quad [1]$$

where $L < 0$ is the loss associated with assigning an employee the skilled task beyond her capacity, and $I_{i,j,t-1}$ is a dummy variable equal to 1 when employee *i* worked at firm *j* in period *t-1*.

In keeping with the standard asymmetric learning literature, we assume that the distribution of skill is such that the unconditional expected value of assigning any time to the skilled task is negative.[17] Therefore, absent any information about an employee, she would only be assigned the unskilled task; this implies (for example) that $\beta_{i,1} = 0$.

Employees are risk neutral and maximize expected utility, which is equal to lifetime wages minus the cost of any effort they expend (we assume a discount rate of zero): $U_i = \sum_t w_{i,t} - \sum_t \beta_{i,t} e$. Maximizing utility rather than income implies that employees may strictly prefer a job that pays the same or even a lesser wage than an alternative if the difference in required effort is large enough (this will be important when we consider pooling equilibria). In other words, when comparing offers, employees consider the wage and assignment pair, not just the wage, and choose the package with the highest utility. We assume that firms are restricted from making negative utility offers.[18]

[17] A common assumption starting with Waldman (1984).

[18] We endogenously derive an outside option for employees due to the presence of a lateral hiring market for experienced workers; it binds when it provides utility that is greater than 0. By assuming employees are risk neutral, all wage contracts are formed in the spot market (i.e., there is no role for insurance via long-term contracts). The restriction on negative utility offers constrains employers from forcing employees out of the firm; this is relevant only under equality mandates, where the presence of lower skilled employers limits the feasible assignments that can be given to higher skilled employees.

B. Timeline, wage formation, and learning

We model two periods of worker production. At the start of each period, a worker is made an offer by her incumbent firm. This offer consists of a wage and an assignment, where the latter is an allocation to the skilled task, with the balance spent on the unskilled task. Each worker then receives an offer from one outside firm, also consisting of a wage and an assignment to the skilled task. The worker switches jobs if doing so strictly increases expected lifetime utility.[19] In the first period, workers are randomly assigned to a firm (which is then the "incumbent"). Since employees are ex-ante identical, the first period offer from the outside firm ensures Bertrand competition obtains for new entrants to the labor market; the outside firm and incumbent make equivalent offers, equal to the lifetime profits that they expect from a new hire (conditional on expected lifetime production, wages, and mobility).[20]

Initially, neither firms nor employees know the employee's type. Following the standard assumption in the asymmetric learning literature starting with Waldman (1984), α_i is revealed to both the employee and to her incumbent firm after one period of work, but not to other firms in the market. Firms will use their knowledge of α_i to inform future wage offers and job assignments.

We assume that precise job assignments cannot be credibly disclosed to outside firms. However, we assume that there is a noisy signal of job assignment, defined as a "promotion". Outside firms can observe if an employee's assignment to the skilled task is above a threshold β'; employees above this threshold are observed to have been promoted, equivalent to having a different title. As is standard in the literature, firms strategically determine which, if any, employees to promote; they maximize their profits by manipulating the information available to outside firms.

We also include an additional signal: when wage disclosure is allowed, employees can make credible disclosure of the incumbent wage offer to the outside firm, from which it will make inferences about the employee's ability when making its offer. Incumbent firms know that their offers will be disclosed and consider the implications of salary disclosure when making

[19] This is equivalent to assuming some infinitesimal switching cost.
[20] An equivalent assumption is that firms compete for new hires, who randomize between identical offers.

wage and job assignments. When a salary history ban is enacted, such disclosure is prohibited, and firms respond accordingly.

Once the worker has selected a firm, she works, wages are paid, and production occurs.

C. Equilibrium wages, mobility, and salary disclosure

We define an equilibrium as a set of wages, job assignments, and mobility decisions that satisfy a Perfect Bayesian Nash equilibrium. Incumbent firms offer their employees a wage-assignment pair of (w_t, β_t). We denote outside offers with a * superscript: outside firms offer prospective lateral hires a wage-assignment pair of (w_t^*, β_t^*).

In the first period, the same wage is offered to all new hires (since firms have no information about employee skill), and we have assumed that they can assign new hires none of the skilled task. Therefore, $(w_1, \beta_1) = (w_1^*, \beta_1^*) = (w_1, 0)$ for some $w_1 \geq 0$.

In the second period, incumbent firms know the skill of their employees, and will vary their wages and assignments accordingly: $(w_2, \beta_2) = (w_2(\alpha), \beta_2(\alpha))$. Under certain circumstances (such as under a salary history ban) these wage and assignment functions will have discontinuities at a specific skill threshold associated with a change in promotion status. In order to align the notation with that used for outside firms, when wages and assignments do depend on promotion status we separately denote $(w_p(\alpha), \beta_p(\alpha))$ for promoted employees and $(w_{np}(\alpha), \beta_{np}(\alpha))$ for employees that are not promoted (note that the domains of the two sets of functions will be non-overlapping). Given that promotion status is only relevant in the second period, we suppress the time subscript.

Outside firms cannot observe α and the wage-assignment pairs they offer to prospective new hires cannot be conditioned on skill. Instead, they are conditioned on salary (if it can be disclosed) and on promotion status. We will see that only one of these sources of information will be relevant in any particular equilibrium. When salary history is relevant, we write $(w_2^*, \beta_2^*) = (w_2^*(w_2), \beta_2^*(w_2))$. When promotion status is relevant, outside firms offer one wage-assignment pair to each promoted employee and another to each employee who is not promoted. We use the shorthand of (w_p^*, β_p^*) for promoted employees and (w_{np}^*, β_{np}^*) for employees who have not been promoted (under a salary history ban, each of these wages and

assignments is a constant, whereas under an equality mandate they can be conditioned on the incumbent wage).

Finally, the mobility decisions are characterized by accepting the wage offered by the incumbent firm or accepting the offer from the outside firm.

A characteristic shared by all the equilibria that we consider is that the outside offers are as attractive as possible, a principle we term *maximal outside offer*. The maximal outside offer is the wage-assignment pair that generates the highest utility affordable for the outside firm conditional on the information available to the outside firm about the employee. [21] As such it would generate zero profits if accepted. To see why this is necessary in equilibrium, consider what happens if the outside firm instead made an offer that would result in positive profits if accepted. The profit-maximizing strategy for the incumbent will be to match that utility, such that it retains the employee at the lowest wage possible. But then the optimal strategy for the outside firm is a slightly better offer (i.e., a higher wage and/or a lower assignment), which would be accepted by the employee and result in positive profits for the outside firm. Of course, this in turn changes the optimal strategy for the incumbent; the only steady state is at the maximal outside offer. We will see that in all cases incumbents will at least match this utility, such that there will be no mobility in equilibrium. The outside firm does not have an incentive to deviate from this strategy, despite the lack of mobility, because lower (and potentially profitable) offers will not be accepted, whereas more attractive offers would be accepted but lead to negative profits.

D. Baseline equilibrium: e = 0

To demonstrate the equilibrium concept and to establish that our framework encompasses the standard approach as a special case, we compute the equilibrium that obtains when effort is costless.

Proposition 1: With costless effort, in the unique equilibrium there exists an inefficient promotion threshold $\alpha' > \beta'$ *and a promotion wage* $w_p^* > 0$ *such that:*

[21] We constrain the outside firm from making offers that would result in negative profits if accepted. This eliminates trivial equilibria, such as those with an extremely high outside offers for employees with a positive wage, forcing the incumbent to pool all employees at a wage of 0.

i) Employees with $\alpha < \alpha'$ are not promoted; incumbents offer a second period wage-assignment pair of $\left(w_{np}(\alpha), \beta_{np}(\alpha)\right) = (0, \min(\alpha, \beta'))$, and outside firms offer $(w_{np}^*, \beta_{np}^*) = (0,0)$;

ii) Employees with $\alpha \geq \alpha'$ are promoted; incumbents offer a second period wage-assignment pair of $\left(w_p(\alpha), \beta_p(\alpha)\right) = \left(w_p^*, \alpha\right)$ and outside firms offer (w_p^*, β_p^*), where $\beta_p^* = \underset{\beta}{\operatorname{argmax}} E[Y_2^* \mid \alpha \geq \alpha']$, the assignment that maximizes the production of employees with skill above the promotion threshold when employed by the outside firm.

No workers change firms, and new entrants to the labor market receive positive wages that reflect the expected lifetime profits from new hires but are not assigned any of the skilled task.
Proof: See Appendix.

The equilibrium in *Proposition 1* has all the characteristics of the standard asymmetric learning models. First, the combination of asymmetric learning and the noisy signal of job assignment results in the classic Waldman result: incumbent firms strategically underpromote high skill employees to frustrate outside offers. Incumbents utilize an inefficiently high promotion threshold because outside firms are severely constrained in the offers they can make to employees who are not promoted. Recall that we assume that the unconditional expectation of assigning an employee to any amount of the skilled task is negative, and the employees who are not promoted have (weakly) less skill than the full population. Therefore, these employees cannot be assigned any of the skilled task by the outside firm, and the maximal outside offer is a wage of 0. Incumbent firms only need match the wage of 0 to retain their employees who are not promoted (when effort is costless, wage and utility are equal), which gives them the incentive to cap the assignment at β' for some employees with skill above that threshold. At a wage of 0, employees that are not promoted are generally paid below their marginal product in the second period and have no returns to skill.

The second feature that matches the standard results is that incumbents may promote the most highly skilled employees if the production gains from the increased assignment outweigh the extra wage they will need to be paid by virtue of their promotion status. In equilibrium, the threshold α' is the skill level at which the profits generated from promoting the employee (and paying the wage w_p^*) equal the profits from not promoting, capping the assignment at β', and paying a wage of 0. In the appendix, we derive the equations that determine the promotion

threshold, and the conditions under which some employees are promoted. The structure of the outside offer is important in this calculation, as is the level of firm-specific human capital. The maximal outside offer is the full expected production that the outside firm can generate from employees knowing only that they have skill above α', which of course excludes any contribution from firm-specific human capital. The optimal assignment may be above α', depending on the distribution of skill, if the gains from better utilizing more skilled employees outweighs the losses from over-assigning employees with skill close to α'.

Incumbent firms must match this wage to retain their employees. Therefore, promoted employees do earn positive wages (and utility). However, they are also generally paid below their marginal product, and wage (and utility) is equalized across all promoted employees regardless of their individual productivity.

Third, by virtue of paying wages below marginal product, incumbents earn positive profits in the second period. This expected profit is paid to employees in the first period, when workers earn wages in excess of their production, as is typical in models with asymmetric learning.

Finally, the equilibrium is inefficient because $\alpha' > \beta'$: there are some under-utilized employees that do not produce at their full potential. The inefficiency results from the noise in the signal. Either a fully informative signal or no signal at all is preferable from an efficiency standpoint, although these would have very different implications for the distribution of wages across employees.

Several policy implications of this baseline equilibrium are worth noting. First, salary disclosure is irrelevant. Wages simply reflect publicly available information (in this case, promotion status), and banning disclosure has no effect on the equilibrium. Second, equality mandates are also irrelevant. At each title, workers are paid identical wages regardless of their skill; this is a standard result in the asymmetric learning literature. Without a cost of effort, there is no intra-title wage variation. The extra production that more skilled employees generate flows directly to firm profits, and thus to the wages paid to new entrants in the labor market.

Finally, although those two policy interventions are irrelevant, there are policy options that would reduce or eliminate the inefficiency caused by strategic underpromotion, including banning disclosure of job history, reducing or eliminating competition for experienced employees, and banning mobility. While some of these are infeasible on their face (e.g., banning

disclosure of job history would invalidate every resume ever written), the point is that such interventions come at no cost, and in fact imply an aggregate gain.[22]

IV. Costly effort, wage disclosure

We now parameterize the model with costly effort. We demonstrate that when employees can disclose their offered wage to outside firms, the unique equilibrium is that of full revelation of employee skill, as if learning were symmetric. Wages are fully informative, noisy signals have no effect on the equilibrium, and the outcome is efficient, in that the full productive capacity of the economy is realized in every period. In other words, the features in the baseline considered above linked to both asymmetric learning and the noisy signal all disappear.

We model wage disclosure as the outside firm being able to observe the wage offer made by the incumbent to the worker: specifically, the outside firm observes w_2 before making its offer of (w_2^*, β_2^*)).[23] Incumbent firms know this and make their offers accordingly. We will see that the wage necessary to induce effort in the skilled task reveals each employee's capacity α to the outside firm, which drives up the wage incumbents must pay. Outside firms are willing to match incumbent offers and assign poached employees to the amount of the skilled task that corresponds to that wage.

Proposition 2: With costly effort and wage disclosure, the unique equilibrium is:

$$(w_1, \beta_1) = (w_1^*, \beta_1^*) = (s, 0)$$
$$(w_2(\alpha), \beta_2(\alpha)) = (\alpha * y, \alpha)$$
$$(w_2^*(w_2), \beta_2^*(w_2)) = (w_2, \frac{w_2}{y})$$

No workers change firms and $s = E[(1-\alpha) * s_0 + \alpha * y * s_1]$*, the expected second period production (and profits) linked to firm-specific human capital. The noisy signal of promotion is irrelevant. The equilibrium is efficient.*

Proof: a) *Existence*. The incumbent's second period wage offer reveals the employee's ability since it is a linear function of α. Outside firms cannot make strictly profitable deviations from

[22] Many models in the literature consider channels other than in-period production through which such interventions could have deleterious effects, such as investments in human capital through education or job training. Our framework highlights that the additional potential effect on production.

[23] As noted above, in the first period this process is equivalent to assuming that firms compete by making simultaneous offers to new entrants to the labor market.

choosing to match this wage ($w_2^*(w_2) = w_2$) and offering the corresponding assignment of $\beta_2^*(w_2) = \frac{w_2}{y}$. This is the maximal outside offer because it fully utilizes the employee and pays a wage equal to her production. Since $y > e$, utility increases with assignment so long as the wage equals the full production; therefore, the outside offer is the highest utility the outside firm can deliver. Decreasing the utility of the offer (with a lower wage and/or a higher assignment) would not attract any lateral hires. Increasing the utility of the offer (via a higher wage and/or lower assignment) would lead to hiring, but negative profits, since the incumbent wage is equal to the maximum production of the employee at the outside firm.

Incumbents cannot make profitable deviations because of the structure of this maximal outside offer. For example, consider an incumbent that offered wages $w_2(\alpha) = \alpha * e$, the minimum necessary to induce effort, resulting in a utility of 0. Any employee with $\alpha > 0$ would accept the outside offer because it pays the same wage but requires less effort, and thus generates positive utility:

$$U^*{}_2(w_2) = \ w^*{}_2(w_2) - e * \beta^*{}_2(w_2) = w_2\left(1 - \frac{e}{y}\right) > 0 \qquad [3]$$

In fact, this outside offer increases the utility for any employee who is paid less than her full (outside) marginal product, because it pays the same wage but requires less effort:

$$U^*{}_2 - \ U_2 > 0 \rightarrow w_2 < \beta_2 * y \qquad [4]$$

In any strategy that is profitable pre-attrition, including pooling and mixed strategies, incremental profits (above those linked to firm-specific human capital) must be generated by employees assigned an amount of the skilled task above $\frac{w_2}{y}$. But those employees accept the outside offer, and employees who are assigned below this level reject it. Therefore, after attrition, any such strategy reduces profits and incumbent firms have no incentive to deviate from paying wages equal to $\beta * y$. Further, because wages equal $\beta * y$ (the marginal the product for outside firms), the only profits that incumbents earn in the second period are due to firm-specific human capital. Firms maximize these profits by assigning employees their full capacity in the skilled task (because $y * s_1 > \ s_0$). These profits are reflected in the initial wage.

b) *Uniqueness*. See Appendix.

Uniqueness is proved by eliminating alternative equilibria. We eliminate equilibria with alternative one-to-one maps of ability to wage by the incumbent by demonstrating that the outside firm can infer ability and profitably poach employees from the incumbent. The

elimination of potential pooling strategies is more complex. The key intuition is that constructing a pool that deters poaching requires the incumbent firm to inefficiently utilize some employees, vis-à-vis alternative assignments that are available to them. However, pool construction and assignments are not observable and the incumbent cannot commit to them. The incumbent deviates from these assignments, invalidating equilibria with deterrent pools.

In equilibrium, then, wages fully reveal employee skill, and employees capture their full (non-firm-specific) marginal product. Wage inequality is high, owing to the high returns to skill. Because salary history is fully informative, any noisy signal about employee ability is irrelevant. Finally, the equilibrium is efficient because each employee is fully utilized.

This result – an efficient equilibrium with fully revealing, marginal product wages – is dependent on costly effort and the disclosure of wage offers. Costly effort forces incumbents to pay their productive employees a non-zero wage, with higher productivity garnering increasingly higher wages. Unlike in Waldman et al., where the incumbent can costlessly pool employees at a wage of zero, with costly effort any pool that deters poaching includes employees who generate strictly lower profits for the incumbent; this creates an incentive for the incumbent to deviate by adjusting the pool, and the pool cannot be an equilibrium. Since pools are not feasible in equilibrium, wage disclosure allows challenger firms to infer ability. Knowing ability, the maximal outside offer is equal to the employee's marginal product at the challenger firm. In fact, outside firms must inquire about the incumbent wage offer, or else they will suffer from adverse selection and earn negative profits.

By itself, this insight is new and our framework can explain why firms do in fact inquire about salary history. Further, from a policy perspective, it is clear that once costly effort is introduced to the framework, neither wage disclosure nor equality mandates remain irrelevant. As discussed above, wage disclosure is central to wage formation. Similarly, in this equilibrium employees with the same title have different wages; wages reflect the continuous distribution of ability rather than the discrete title thresholds. This suggests that equality mandates, which seek to compress the distribution of wages within titles, could restrict the ability of firms to take advantage of within-title differences in employee ability. We explore these concepts below.

V. Salary history bans and equality mandates

Armed with this framework, we now analyze how various interventions in the labor market targeting wage disclosure and wage formation affect task assignments, wages, and efficiency.

A. Salary history bans

A salary history ban prohibits the outside firm from inquiring about an employees' incumbent wage. With salary histories suppressed, the outside firm cannot use the second period wage offer to infer worker type; it must instead rely solely on the noisy promotion signal. Therefore, incumbent firms can mask the assignments of their employees, up to that threshold. The result is similar to *Proposition 1*: incumbent firms under-promote employees in order to reduce their outside option and allow them to pay the minimum wage possible.

Proposition 3: With a salary history ban, the unique equilibrium has an inefficient promotion threshold $\alpha' > \beta'$ *and a promotion wage* $w_p^* > 0$ *such that:*

i) Employees with $\alpha < \alpha'$ *are not promoted; incumbents offer a second period wage-assignment pair of* $\left(w_{np}(\alpha), \beta_{np}(\alpha)\right) = (e * \min(\alpha, \beta'), \min(\alpha, \beta'))$*, and outside firms offer* $(w_{np}^*, \beta_{np}^*) = (0,0)$*;*

ii) Employees with $\alpha \geq \alpha'$ *are promoted; incumbents offer a second period wage-assignment pair of* $\left(w_p(\alpha), \beta_p(\alpha)\right) = \left((w_p^* - e * \beta_p^* + e * \alpha,\ \alpha\right)$ *and outside firms offer* (w_p^*, β_p^*)*, where* $\beta_p^* = \underset{\beta}{\operatorname{argmax}}\, E[Y_{i,2}^* - \beta * e \mid \alpha \geq \alpha']$*, the assignment that maximizes the utility of employees with skill above the promotion threshold when employed by the outside firm.*

No workers change firms, and new entrants to the labor market receive positive wages that reflect the expected lifetime profits from new hires but are not assigned any of the skilled task.

Proof: See appendix.

The proof follows that of *Proposition 1* closely, as the equilibria are very similar. The main difference is the need to compensate for the disutility of effort. For example, the flat incumbent wage of 0 from *Proposition 1* paid to employees that are not promoted is replaced with the minimum wage necessary to ensure utility of 0: employees who are not promoted are

paid wages that just compensate for their effort. Therefore, wage (but not utility) of these employees increases with skill.

Similarly, because of the cost of effort, outside firms no longer compete for workers purely on the basis of the wage; they offer utility, equal to wage less the cost of effort. Knowing the promotion threshold, the maximal outside offer is the greatest utility outside firms can offer to promoted employees and still earn nonnegative profits (rather than simply the maximum wage, as in *Proposition 1*). Incumbent firms must match that utility to retain their employees. Therefore, all promoted employees also earn equivalent utility (equal to that offered by the outside firm); because they have differential assignments based on their skill, their wages also increase with skill. Their utility is above that of employees who are not promoted, just as in *Proposition 1*.

Firms earn profits in the second period, as they generally pay wages below marginal product, although the profits are lower than was the case in *Proposition 1* because second period wages must compensate for effort. The profits are reflected in first period wages.

We conclude that jamming the wage signal introduces noise into the learning process. Because they are used to motivate effort, wages function as the ultimate signal; without them, outside firms are forced to rely on promotion signals. Incumbent firms strategically adjust the promotion threshold to maximize their profits; this limits the ability of outside firms to make lateral hires.

Effectively, banning wage disclosure increases the bargaining power that firms have over their more experienced and skilled employees. As a result, they earn wages below their potential (outside) marginal product. The benefits are dispersed across the full population in the form of higher initial wages. This is a net benefit to low skilled employees, who would not have been rewarded with high wages if salary disclosure was allowed. In other words, compared to the case where wage disclosure is allowed, wage inequality is low (as stated above, it is not zero because the more skilled workers must be compensated for their increased effort expenditure). However desirable the reduction in wage (and utility) inequality may be, it comes at the cost of production inefficiencies due to the under-assignment of high skill employees.

B. Equality mandates

We model an equality mandate as a requirement that firms pay all employees with the same title the same wage. In other words, firms must pay one wage (w_{np}) to all employees who are assigned below the observable promotion threshold β', and another to those that are "promoted" past that threshold (w_p). While this is not the explicit form that rules such as pay transparency laws take, the intent of such rules is to compress the distribution of wages across employees, and there is reason to believe that this is their ultimate effect.[24]

The constraint imposed by an equality mandate is very different from the constraint imposed by a salary history ban. A ban reduces the information available to an outside firm, and thus reduces the outside option of the more skilled employees, with the benefits accruing to the incumbent firm (in the second period) and to less skilled employees (in the first period). In equilibrium, costly effort still leads to within-title salary differentiation, as the incumbent differentiates assignments within titles and so must pay each employee a wage that just covers her effort. In contrast, an equality mandate binds the incumbent firm. As we will see, costly effort implies that a fixed wage limits the range of possible assignments the incumbent can utilize.

The limitation on wages implies that the equilibrium takes the familiar form: a promotion threshold α', such that employees with skill above α' are promoted, because it is worth paying them the higher wage. If there is an employee with skill sufficiently high to promote and pay the wage w_p, then any employee with skill above that threshold must also be promoted, since they can be given at least the same assignment. We will demonstrate that such an equilibrium must

[24] This analysis applies to equality-based initiatives that make it costly to pay differential wages based on unverifiable characteristics such as skill or productivity. Requiring publication of wages in job postings, for instance, pre-commits the employer to pay within the wage band and limits discretion to base wages on productivity. Pay transparency laws make it easier to discover and allege the impropriety of pay disparities, and both new and existing laws make such disparities costly. For instance, a recent 9th Circuit opinion interpreted the Equal Pay Act to required employers to prove that pay differentials are based upon valid factors such as productivity as opposed to sex, and disallowed the use of salary histories as such a valid factor; the minority dissent noted that this rule would "demand a lockstep pay system." Rizo v. Yovino, Case No. 16-15372, 9th Circuit, February 27, 2020. Pay equity advocacy groups also recommend practices such as lockstep compensation, limiting managerial discretion in setting pay, standardizing compensation, and creating a culture of equality. See California Commission on the Status of Women and Girls, What Can I do to Promote a Culture of Pay Equity?, https://women.ca.gov/californiapayequity/employers-resources/what-can-i-do-to-promote-a-culture-of-pay-equity/.

have several other characteristics of economic interest (in the appendix we discuss the conditions under which such equilibria exist).

The incumbent will choose wages w_p and w_{np}, the promotion threshold α', and assignment functions $\beta_p(\alpha)$ and $\beta_{np}(\alpha)$, that maximize its profits given the outside offers. Outside firms make offers that are conditioned on both promotion status and wages: $\left(w_p^*, \beta_p^*\right) = \left(w_p^*(w_p), \beta_p^*(w_p)\right)$ and $\left(w_{np}^*, \beta_{np}^*\right) = \left(w_{np}^*(w_{np}), \beta_{np}^*(w_{np})\right)$. There are two types of poaching strategies that an outside firm may attempt. The first is the familiar one: an employee assigned more than $\frac{w_2}{y}$ by the incumbent can be poached by the outside firm. The second is a consequence of this limitation on assignments: if the incumbent "bunches" employees of high skill at an assignment of $\frac{w_2}{y}$, it is possible that the outside firm can profitably assign this group of employees an assignment of $\frac{w_2}{y} + \varepsilon$, if the losses from any employees that would be over-assigned (such as those with $\alpha = \frac{w_2}{y}$) are outweighed by the gains from better utilizing those employees with skill above $\frac{w_2}{y}$.

Equilibrium requires that neither type of firm can increase its profits by varying these offers, conditional on the strategy being deployed by the other type of firm. In particular, the incumbent strategy must be robust to both poaching strategies.

We derive a series of constraints on the equilibria, to enable us to characterize the effect of an equality mandate. The first set of constraints involves mobility; unsurprisingly, there cannot be mobility in equilibrium.

Lemma 1: There can be no mobility amongst the set of employees who is not promoted.

Proof: Any mobility would start at the least productive employees who were not promoted, which includes those with skill of 0 (who clearly have skill below the promotion threshold). However, no such set of employees can be profitably offered positive utility by the outside firm. QED

Knowing that there cannot be mobility for employees who are not promoted implies a lower bound of the profitability of those employees.

Lemma 2: The minimum average profitability of the employees who are not promoted is s_0.

Proof: The incumbent can assign none of the skilled task and pay a wage of 0 to employees who are not promoted. Outside firms cannot poach under this strategy as they cannot offer positive utility (similarly to the logic above). This strategy generates profits of s_0 per employee. Therefore, this is the minimum average profitability of those employees in equilibrium. QED

The fact that the employees who are not promoted generate a profit in equilibrium constrains the outcomes for the employees who are promoted.

Lemma 3: There can be no mobility for employees who are promoted.

Proof: The incumbent cannot commit to the promotion criteria. A promoted employee that accepted the outside offer generates no profits for the incumbent, who would instead reassign her such that she was not promoted and generates positive profits. QED

The lack of mobility in equilibrium further implies that employees with the greatest assignment to the skilled task within each title earn a utility equal to the utility of the outside offer at that wage (with mobility it is possible that they had utility below the outside offer and accepted it). If the employee with the greatest assignment had utility above the outside offer the incumbent could raise its profits by increasing the assignment of those employees, and/or reducing the wage.

With that insight in hand, we can further constrain the form any equilibrium must take.

Proposition 4: Under an equality mandate, the equilibrium wages and assignments of employees who are not promoted are bound above:

i) $w_{np} \in [0, s_0]$

ii) $\beta_{np}(\alpha) \in [0, s_0/y]$

Proof: See Appendix.

Consider first the upper bound on the wage w_{np}. It cannot be above s_0 because then employees with $\alpha = 0$ are unprofitable for the incumbent, whose production is capped at s_0. Rather retain employees that reduce profits, the incumbent would instead assign those employees enough of the skilled task that they accept the outside offer. Any assignment above β_{np}^* would suffice. Outside firms earn negative profits from these employees at any positive wage; thus, they would reduce their offer (and/or increase the assignment β_{np}^*), breaking the equilibrium. The upper bound on assignments detailed in Part (ii) follows directly from the maximal outside offer. As in the other equilibria we consider, any employee assigned more than w_{np}/y of the

skilled task can be poached by the outside firm, which can make an offer of $(w_{np}, \frac{w_{np}}{y} + \varepsilon)$ that is utility-improving for those employees at some $\varepsilon > 0$ and generates profits of $\varepsilon * y$ for the outside firm. Given the result in Part (i), this caps the assignment at s_0/y.

This is an important constraint imposed by the equality mandate. By virtue of needing to pay the lowest skilled employees the same wages as their higher skilled (but not promoted) colleagues, wages are capped, which translates into an upper bound on possible assignments. This introduces a new form of efficiency loss: within-title underutilization. Some employees who are not promoted cannot be fully utilized because an assignment equal to their skill would reduce their utility and expose them to poaching. Firms cannot in general afford to pay wages sufficiently high to protect all employees when fully utilized because that generates losses from overpaying the lower skilled employees.

Equality mandates impose a similar constraint on employees who are promoted.

Proposition 5: Under an equality mandate, the equilibrium assignments made by incumbents to employees who are promoted are bound above and below:

i) $\beta_p(\alpha) \leq max\ (\frac{w_p}{y}, \beta')$

ii) $\beta_p(\alpha) \geq max[\beta', (w_p + w_{np} * K)/C]$

where $K = s_1 + s_0/y$ *and* $C = y * (1 + s_1) - s_0$

Proof: See Appendix.

The upper bound on the assignment is derived in the same way as the upper bound in *Proposition 4*. In general, at any assignment above w_p/y the employee can be profitably poached by the outside firm. The only exception is if the max assignment is equal to β'. As we will see below, it is possible that some employees are over-assigned; if that occurs and the maximum assignment is equal to β', then the outside firm cannot offer a wage as high as $y * \beta'$ to the promoted employees, and thus the incumbent need only match the lower wage.

There are two components to the lower bound. First, by definition, promotion requires that the assignment must be greater than β'. Second, the profits generated by the least skilled promoted employee must at least equal the profit that she would generate if not promoted (and thus paid w_{np}) and given the maximum assignment outlined in *Proposition 4*. The lower bound on $\beta_p(\alpha)$ is an assignment level that generates that profit, given the equilibrium wages.

There are several implications of *Proposition 5*. First, in general (i.e., except when $w_p = y$), the most skilled promoted employees are underutilized. This is the analogous result to the underutilization of the most skilled employees who are not promoted. The wage necessary to retain the most skilled employees when assigned up to their capacity is not affordable because it reduces the profits generated from the lower skilled but still promoted employees. That leads to the following general conclusions regarding the effect of equality mandates.

Corollary 1: Equality mandates lead to efficiency losses from within-title underutilization.

An interesting implication is that this underutilization potentially gives employers an incentive to force out their least skilled workers. Recall that we assume that firms cannot offer negative utility wage-assignment pairs to their employees. In all other settings we consider, this assumption does not impose a binding constraint; firm-specific human capital implies that it is worthwhile employing even those workers with $\alpha = 0$. That potentially changes under an equality mandate. While the least skilled workers still generate production from firm specific human capital, their presence in the firm limits the equilibrium assignments (and thus production) of the more skilled workers. This drag could outweigh their production, such that the net effect of the lower skilled employees on profits is negative. One possible effect of equality mandates is that firms will seek to "weed out" productive but low skilled workers, possibly through the strategic use of unappealing assignments.

Another possible implication is that firms will have an incentive to create more titles. With more titles, the incumbent firm reduces its inefficient underutilization by creating more ranges of feasible assignments, and thus more fully realizes the profits from firm-specific human capital. This distinguishes an equality mandate from a salary history ban, where a relative paucity of titles better obscures the true ability of the incumbent employees and thus better frustrates outside offers.

Corollary 2: Equality mandates lead to lower average wages and productivity.

This conclusion follows directly from the underutilization within each title; overall productivity and thus average wages must fall under an equality mandate. Like a salary history ban, the efficiency losses translate into reduced utility for some skilled employees. Unlike a ban, utility is reduced because the employees are not fully utilized, and thus their wage cannot reflect their full potential production, rather than because their wage is below their full production. Any

underutilized employee has a wage that reflects her full realized production, excepting that linked to firm-specific human capital; this is ensured by the maximal outside offer. Their utility loss is driven by the fact that utility is highest when both fully utilized and fully paid. In other words, an equality mandate does not reduce the outside option available to an employee conditional on her assignment (i.e., she is still "fully paid"). It restricts the assignment, which in turn restricts her ability to fully realize her potential. As a result, the losses to more skilled employees are not reflected in higher salaries to new entrants to the labor market. In fact, starting salaries likely fall, because employees who are not underutilized are overpaid. They receive wages above their (outside) marginal product because they are paid just like more skilled employees with the same title.

A final implication of *Proposition 5* is that equality mandates raise the prospect of over-assignment. The possibility arises because of the significant wedge between the assignments that can be given to employees at each title. The range of possible assignments for employees who are not promoted is quite narrow, and close to 0 (particularly when s_0 is small). This implies that employees with skill just below β' are severely underutilized if not promoted. Promoting these employees allows for much greater utilization of their skill, and it is possible doing so increases profits, if the gains from more fully utilizing them, net of the losses linked to over-assignment, outweigh the extra wage that accompanies promotion.

An example helps illustrate the point. Assume that the promotion threshold β' is high at 0.95 but that $s_0 \approx 0$. With such a small s_0, the maximum assignment for employees who are not promoted ($\beta_{np}(\alpha) = s_0/y$) is also close to 0. Therefore, all employees who are not promoted generate (nearly) no profits. In this sense, an employee with a skill level of, say, $\alpha = 0.9$, is severely underutilized when not promoted. However, this employee can generate positive profits if promoted, so long as the wage w_p and the loss from over-assignment L are small relative to the production associated with such a high skill level:

$$0.9 * y * (1 + s_1) + 0.05 * L - w_p > 0 \rightarrow$$
$$w_p - 0.05 * L < 0.9 * y * (1 + s_1) \qquad [5]$$

It is only feasible to over-assign employees with skill below the public promotion threshold β'; these employees are the only ones for which over-assignment is ever necessary. An employee with skill above β' can be fully utilized without being over-assigned. In fact, increasing the assignment of an employee with skill above β' has the counterproductive effect of

reducing her utility (and thus making her easier to poach) and reducing firm profits. As a result, over-assignment only occurs when the minimum assignment for promoted employees is pegged at the public threshold β'.

Corollary 3: In general, the threshold α' is inefficient. When $\beta' < (w_p + w_{np} * K)/C$, with C and K defined as in Proposition 5, firms under-promote in the classic Waldman sense. When $\beta' > (w_p + w_{np} * K)/C$, some employees with skill below β' are over-assigned.

Although the above analysis details the key features necessary to describe the implications of an equality mandate, for completeness we outline an example equilibrium in the appendix, along with the conditions under which it obtains. This requires detailing the off-equilibrium outside offers that keep the incumbent from deviating to alternative wages.

VI. Discussion

In this article, we have demonstrated that the addition of costly effort into the standard asymmetric learning framework makes the wage perfectly informative. This occurs because employers must pay high ability workers a greater wage to induce the optimal level of effort. We have shown that this addition prevents strategic manipulation of noisy public signals by underpromotion, which is a standard feature and source of inefficiency in asymmetric learning models. In fact, the signaling value of wages is so powerful that wage disclosure leads to an equilibrium equivalent to one of symmetric learning: firms pay their employees their full marginal product each period, even absent other public signals, and the full productive capacity of the economy is utilized each period.

By modeling wages as an input into production, in keeping with the basic fundamentals of contract theory, our framework is better equipped than the standard ones for evaluating policies that intervene with regard to wage formation, competition, or wage signal suppression (as with salary history bans). In contrast, the standard employer learning models describe wages as an output: wages reflect the signals available to the market and the corresponding strategic responses to those signals. These standard models lead to a paradoxical result, in that a large class of seemingly drastic interventions in the labor market are either costless or even welfare improving, such as banning competition for workers or mandating equal wages for all workers. Our model also explains why many employers ask about wages in the hiring process when they are allowed to do so.

Our framework allows us to assess the implications of suppressing the wage signal, as occurs with salary history bans, or in artificially compressing wages, as occurs with various wage equality initiatives. Suppressing the wage signal with a ban introduces inefficient reliance upon noisy signals. Aggregate welfare, the returns to skill, and ex-post wage inequality are all reduced. This last feature may be salutary, though it comes at some cost. Our model also indicates that there are significant distributional effects across workers: high skill workers lose because their outside option is reduced when they can no longer use their higher wage to signal their ability to outside firms, while low skill workers benefit as the profits from this shift in bargaining power are distributed to new hires. This is consistent with the emerging empirical literature on salary history bans, which finds conflicting results across population subsamples. This literature (Agan, Cowgill & Gee (2020, 2021), Bessen, Meng & Denk (2020), Hansen & McNichols (2020), Mask (2020), Sinha (2019), (Sinha (2022)) is roughly consistent with our prediction that salary history bans will benefit those with less-developed career skills, given the significant data limitations.

Wage equality initiatives include measures such as mandating disclosure of internal pay disparities, requiring salary ranges in job listings, and legal rules that place the onus on employers to justify pay disparities. We posit that such measures generally serve to make pay disparities costly, and that the effect is to standardize wages around observable, verifiable, non-suspect traits such as job titles, which leads to lock-step compensation within job titles. While this is not the only possible way to conceptualize such measures, it does accord with the intent of the policy enactors and statements of equality activists. Our model shows that constraining wages by title leads to production inefficiencies and lower wages: firms that must pay employees of different skill levels the same wage must also assign them (approximately) the same level of the skilled task in order to retain them, due to the cost of effort involved in higher assignments. This results in at least some under-assignment of the higher skilled employees within each title band, and outside firms are unable to discern employee skill level. Production is lower and average wages are lower, although there are distributional transfers from higher to less skilled workers. These results are consistent with the emerging empirical literature on pay transparency laws, which find that highlighting internal disparities leads to more equal pay but overall lower wages (Cullen 2023).

An interesting aspect of equality mandates is that they may have differential effects among certain groups, depending on the particular mechanism at play. While a general lockstep policy would be expected to have a uniform effect among employees, obscuring ability equally, pay equality laws targeting suspect classifications (such as pay disparity by sex) may have heterogeneous effects. For example, consider a firm employing one male and one female at the same title, where females are a protected class and males are not; if the female worker is of higher ability, there would be no constraint on pay. On the other hand, were the male to be of higher ability, the firm would be constrained to pay each employee the same by reducing the male's salary, increasing the female's salary, or a combination of both. Hence, we would predict such a rule to obscure the ability of high ability males and low ability females, with different consequences for employee mobility and returns to human capital among those groups.

This model also allows several additional avenues of investigation for future work. Our model can, for instance, accommodate employer bias against certain groups in order to investigate the effect of equality initiatives in the context of gender or race-based discrimination. Preliminary work by the authors (Meli & Spindler 2019) along these lines indicates that such initiatives can lead to lower pay disparities between groups, but that significant distributional effects work to the disadvantage of high-ability workers and benefit low ability workers. This has important implications for the returns to human capital and the incentives to acquire it. An additional use of our model is in the role of employee mobility in mitigating the effects of discrimination: discriminated-against employees may switch to non-discriminatory employers. Again, preliminary work by the authors indicates that salary history bans, as well as other steps that may impede employee mobility, can work to the detriment of discriminated-against high ability workers.

VII. Conclusion

In this article, we have presented a parsimonious model of wage formation and employee mobility that can be used to analyze equality-minded reforms. By including costly effort in the standard asymmetric learning model, wages become fully informative of employee ability: assignment to a greater amount of the high skill task lowers employee utility, which makes such an employee easier to poach unless offset by a correspondingly higher wage. Higher wages reveal employee ability, generating competition with outside firms, which leads to full marginal

product wages and optimal economic assignments in equilibrium. Wages are thus an important signal in conveying employee ability to outside firms, and disclosure of salary histories is economically efficient in this context. Our result stands in contrast to the standard asymmetric learning literature, in which outside employers rely upon noisy promotion signals to discern employee ability, leading to strategic underpromotion by incumbent employers.

Our model may also be used to analyze reforms and recent laws that aim to redress pay disparities among various groups. In the context of our model, we interpret these reforms to either suppress the wage signal (as is the case with salary history bans) or to constrain the employer's ability to base pay on non-verifiable ability levels (as with lockstep compensation regimes imposed by pay equity laws, mandatory wage range postings, and mandatory disclosure of internal pay disparities). Suppressing the wage signal with a salary history ban causes a return to the standard asymmetric learning equilibrium, in which outside firms rely upon noisy promotion signals. Equality mandates force employers to pool wages by the verifiable promotion signal, which in turn requires them to pool assignment levels in order to prevent poaching of higher ability employees. This leads to underutilization of high-ability employees, lower production, and lower average wages. With both bans and equality mandates, there are distributional effects between high and low ability workers, in which returns to human capital are generally decreased. Our results are broadly consistent with the emerging empirical literature analyzing these classes of reforms.

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Appendix

Proposition 1: With costless effort, in the unique equilibrium there exists an inefficient promotion threshold $\alpha' > \beta'$ *and a promotion wage* $w_p^* > 0$ *such that:*

i) *Employees with* $\alpha < \alpha'$ *are not promoted; incumbents offer a second period wage-assignment pair of* $(w_{np}, \beta(\alpha)) = (0, \min(\alpha, \beta'))$*, and outside firms offer* $(w_{np}^*, \beta_{np}^*) = (0,0)$

ii) *Employees with* $\alpha \geq \alpha'$ *are promoted; incumbents offer a second period wage-assignment pair of* $(w_p, \beta(\alpha)) = (w_p^*, \alpha)$ *and outside firms offer* (w_p^*, β_p^*)*, where* $\beta_p^* = \underset{\beta}{\operatorname{argmax}} E[Y_2^* \mid \alpha \geq \alpha']$*, the assignment that maximizes the production of employees with skill above the promotion threshold when employed by the outside firm*

No workers change firms, and new entrants to the labor market receive positive wages that reflect the expected lifetime profits from new hires, but are not assigned any of the skilled task.

Proof: We assume that the equilibrium is characterized by a promotion threshold α', and derive the conditions under which is it in fact an equilibrium. We start with the pay and assignments of employees that are not promoted. Because we assume that the unconditional expected value of assigning an employee to any of the skilled task is negative, employees who are not promoted can only receive offers $(w_{np}^*, \beta_{np}^*) = (0,0)$ from the outside firm (this population is of lower skill that the full population). With the outside offer capped at 0, the incumbent need only pay these employees a wage of 0. At that wage, the incumbent maximizes its profits by fully utilizing the capacity of these employees, up to the public promotion threshold. Therefore, it assigns these employees $\beta(\alpha) = \min(\alpha, \beta')$. Neither incumbents nor

outside firms have an incentive to deviate. We conclude that, with a given threshold α', the wages and assignments detailed in Part (i) meet the criteria for an equilibrium.

Next we turn to the threshold α' and the wages and assignments for promoted employees. We first consider the optimal wage and assignment strategy for the incumbent conditional on the outside offer w_p^*. In order to retain promoted employees, incumbent firms must match that wage. Although we derive the exact form of w_p^* below, for now it is enough to realize that $w_p^* > 0$. Outside firms know that they can assign new hires at least α' of the skilled task (this is a minimum because it is possible that there is a greater level of assignment that increases the average production), and that these hires will generate average production of at least $\alpha' * y$. If the prevailing wage for these employees was 0, outside firms would have an incentive to deviate to some small positive wage, hire those employees, and earn positive profits.

This positive wage creates a wedge between $\alpha'$and the public promotion threshold β'. From the above analysis, the maximum profit generated by an employee who is not promoted is $\beta' * y * (1 + s_1) + (1 - \beta') * s_0$. The incumbent will only promote employees if doing so generates profits at least that high. But promoting entails a positive wage. At the threshold α', the incumbent must be indifferent between promoting (and paying a wage of w_p^*) and not promoting (and paying a wage of 0). We compare the profits from promotion and no promotion to compute the relationship between α' and β':

$$\alpha' * y * (1 + s_1) + (1 - \alpha') * s_0 - w_p^* = \beta' * y * (1 + s_1) + (1 - \beta') * s_0 \rightarrow$$

$$\alpha' = \beta' + w_p^*/K \qquad \text{[A1]}$$

where $K = y * (1 + s_1) - s_0 > 0$. First, it is clear from [A1] that $\alpha' > \beta'$. Second, with a fixed wage but production that increases with assignment (up to ability), incumbent profits are maximized by fully utilizing all employees with skill greater than α'. Therefore, in equilibrium incumbents offer promoted employees $(w_p, \beta(\alpha)) = (w_p^*, \alpha)$.

We now turn to the outside firm, which simultaneously solves its own maximization problem, choosing the wage w_p^* conditional on the promotion threshold α'. Knowing that only employees with $\alpha > \alpha'$ are promoted, the outside firm can determine β_p^*, the optimal assignment for promoted employees:

$$\beta_p^* = \underset{\beta}{\operatorname{argmax}}\, E[Y_2^* \mid \alpha \geq \alpha'] \qquad \text{[A2]}$$

It is possible that $\beta_p^* = \alpha'$. It is also possible that $\beta_p^* > \alpha'$; some over-assignment of the lowest skilled promoted employees is optimal if the gains from better utilizing the more skilled employees outweigh the losses from over-assignment of the less skilled employees.

Outside firms cannot offer wages above the production of promoted employees at that optimal assignment (doing so would guarantee negative profits). Therefore, $w_p^* \leq E[Y_2^* | \beta = \beta_p^*, \alpha \geq \alpha']$. However, in equilibrium the incumbent will match the outside offer, which implies that it also cannot be the case that $w_p^* < E[Y_2^* | \beta = \beta_p^*, \alpha \geq \alpha']$. This would give the outside firm an incentive to deviate to a wage of $w_p^* + \varepsilon$ for some $\varepsilon > 0$, which would be accepted by all promoted employees and generates profits of $E[Y_2^* | \beta = \beta_p^*, \alpha \geq \alpha'] - (w_p^* + \varepsilon)$. The increment ε can always be chosen small enough that this sum is positive. We conclude that:

$$w_p^* = E[Y_2^* | \beta = \beta_p^*, \alpha \geq \alpha'] \qquad \text{[A3]}$$

Together, [A1] and [A3] give two relationships between α' and w_p^*. Each can be used to express w_p^* as an increasing function of α'. In the case of [A1], this is a linear function. For [A3], it is not necessarily linear, but we do know that $w_p^* \geq \alpha' * y$: the outside firm can at least assign lateral hires an amount of the skilled task equal to the minimum skill of the set of promoted employees. Further, when $\alpha' = 1$, $w_p^* = y$: if only the most skilled employees are promoted, outside firms make the highest possible offer.

The unique equilibrium is defined by the crossing of the functions [A1] and [A3] in the feasible domain for α' ($\beta' < \alpha' \leq 1$). Evaluated at β', the function in [A1] yields $w_p^* = 0$, which is below the level of [A3], which requires that $w_p^* \geq \beta' * y$. Therefore, these functions cross in this domain iff [A1], evaluated at $\alpha' = 1$, results in $w_p^* \geq y$. Then the linear [A1] has at least one point in the domain of α' that is weakly greater than the curve in [A3], and thus the two (increasing) functions must cross. We rewrite [A1] to express w_p^* as a function of α' and solve for the parameter set that ensures a crossing:

$$w_p^*(\alpha') | \alpha' = 1 \geq y \rightarrow$$

$$(1 - \beta') * K \geq y \rightarrow$$

$$\beta' \leq (y * s_1 - s_0)/[y * (1 + s_1) - s_0] \qquad \text{[A4]}$$

Note that [A4] is expressed as an upper bound on the public promotion threshold β'. If the public threshold is low, then high skilled employees are severely underutilized when not promoted, and thus there is more scope to promote them despite the higher salary that promotion

requires. If the public promotion threshold is very high, then there is little additional production to be gained from fully utilizing highly skilled employees, and it is possible that there is no crossing that defines an equilibrium. If there is no crossing, then in equilibrium no employees are promoted (i.e., all employees with skill above β' have assignments capped at β'), which is equivalent to $\alpha' > 1$.

Regardless of the exact level of α', at least some employees are inefficiently utilized. This results in positive second period profits for the incumbent firm, which are reflected in the first period wages. QED.

Proposition 2 (uniqueness): With costly effort and wage disclosure, the following equilibrium is unique:

$$(w_1, \beta_1) = (w_1^*, \beta_1^*) = (s, 0)$$
$$(w_2(\alpha), \beta_2(\alpha)) = (\alpha * y, \alpha)$$
$$(w_2^*(w_2), \beta_2^*(w_2)) = (w_2, \frac{w_2}{y})$$

Where no workers change firms and $s = E[(1-\alpha) * s_0 + \alpha * y * s_1]$.

Proof: We demonstrate uniqueness by eliminating potential alternative equilibria. First, any 1-to-1 mapping of ability to wage by the incumbent is revealing of α. If the incumbent wage was below $\alpha * y$, the outside firm could profitably poach the employee by making an offer between the incumbent wage and $\alpha * y$. This would reduce incumbent profits vis-a-vis the equilibrium, because of lost production linked to firm specific human capital. Therefore, no other 1-1 mapping is a viable equilibrium.

Second, we consider pooling strategies, in which the incumbent pays employees of different skill levels the same wage. To describe a potential pooling equilibrium, we must characterize the pools as constructed, paid, and assigned by the incumbent, and the offers made by the outside firm. The incumbent pooling strategy is defined by a set of triples $[w_2, \lambda(\alpha), \beta(\alpha)]$, where w_2 denotes the wage paid to employees in the particular pool, $\lambda(\alpha)$ denotes the measure of employees of ability α included in the pool, and $\beta(\alpha)$ denotes the assignment to the skilled task to each member of the pool (the incumbent can tie assignments to ability).[25] Every

[25] Note that $\beta(\alpha)$ need not be 1-1. The triples must be defined for all feasible levels of w_2, i.e., those between 0 and y.

employee must be assigned to a pool, there is no requirement that all pools have multiple skill levels, and for some feasible wages $\lambda(\alpha)$ may be the null set.

Outside firms cannot observe $\lambda(\alpha)$ or $\beta(\alpha)$, and the incumbent cannot commit to them. Therefore, the incumbent will deviate from $\lambda(\alpha)$ and/or $\beta(\alpha)$ in any pool if it is profitable to do so. These deviations in pool construction and assignment will prove critical to invalidating prospective pooling equilibrium.

The strategy of the outside firm is characterized by a set of wages and assignments that are functions of the incumbent wage: $(w_2^*(w_2), \beta_2^*(w_2))$. This needs to be specified for all feasible incumbent wages, to ensure that the incumbent cannot deviate from a proposed equilibrium.

First, we show that there can be no mobility in equilibrium. The incumbent always has an incentive to pay its employees with ability of 0 a wage of 0. The outside firm cannot poach these employees and they generate incumbent profits of s_0. Therefore, in equilibrium, it must be that $(w_2^*(0), \beta_2^*(0)) = (0,0)$. Any employee that accepts an outside offer generates no profits for the incumbent. The incumbent would increase its profits from that employee to s_0 by reassigning the employee to $(0,0)$. Therefore, a viable pooling equilibrium must have no mobility.

The lack of mobility has several implications for any prospective pooling equilibria. First, without mobility, incumbents will adjust $\lambda(\alpha)$ and $\beta(\alpha)$ within each pool such that no employee is over-assigned to the skilled task (i.e., $\beta(\alpha) \leq \alpha$ for all α with $\lambda(\alpha) > 0$). This must be true because, without mobility, an employee that is over-assigned reduces profitability from the losses associated with over-assignment (if there were mobility, then it is possible that an incumbent will not realize the losses from over-assignment). The incumbent increases profits by reducing the employee's assignment. This increases the utility of the employee, by requiring less effort, and so reassignment does not affect the assumption of no mobility.

Second, if there is a pool in which the profits associated with a given skill α are above the profits employees with that skill would generate in all other pools, then it must be the case that $\lambda(\alpha) = f(\alpha)$ for that pool, and $\lambda(\alpha) = 0$ for all other pools. For example, all employees with skill of 0 will be assigned a wage and assignment of 0. In any other pool they generate lower profits.

A third implication is that the maximal outside offer places an upper bound on the assignments that can be given to pooled employees. The outside offer is a wage and assignment

pair based on the incumbent wage: $(w_2^*(w_2), \beta_2^*(w_2))$. Since the outside firm cannot distinguish between pooled employees, this offer must be made to every member of the pool, although the outside firm knows that only employees for whom this offer is utility-improving will accept it. Knowing that no employee is over-assigned, the minimum equilibrium outside offer is $(w_2^*, \beta_2^*) = (w_2, \frac{w_2}{y})$. As above, any offer with a utility below this cannot be a stable point. At any less attractive offer, the incumbent would include in the pool some employees $\alpha > \frac{w_2}{y}$ and fully utilize them. But then the outside firm could poach those employees with an offer of the form $(w_2^*, \beta_2^*) = (w_2, \frac{w_2}{y} + \varepsilon)$, and generate a positive profit of $\varepsilon * y$ (they were not over-assigned at the incumbent, and so generate production of $w_2 + \varepsilon * y$).[26] Note that this holds for pools with a null set of employees: the outside offer must be $(w_2, \frac{w_2}{y})$. If this were not the case, then the incumbent would deviate and profitably construct a pool at that wage.

If the outside offer takes the form $(w_2^*, \beta_2^*) = (w_2, \frac{w_2}{y})$ for all feasible levels of w_2, then the incumbent maximizes profits by assigning each employee up to her ability and paying a wage equal to $\alpha * y$. In other words, it would set $\lambda(\alpha) = 0$ in every pool except that with $w_2 = \alpha * y$ and would set $\beta(\alpha) = \frac{w_2}{y}$, effectively dissolving any pool. Doing so maximizes profits because the only profits are driven by firm specific human capital: employees are paid wages equal to their realized outside marginal product. We assume that $y * s_1 > s_0$, and thus the profits linked to firm-specific human capital are highest when employees are fully utilized. A lower wage for any employee would require a lesser assignment, and thus lower profits.

However, it is possible that the maximal outside offer is even more attractive. Consider a pool with $w_2 > 0$ that includes employees with skill above w_2/y, who are under-assigned. It is possible that the outside firm can increase the utility of its offer by assigning more than w_2/y, if the losses from over-assigning those employees with skill at or below w_2/y who will accept the offer are outweighed by the gains from better utilizing employees with skill above w_2/y. In equilibrium, the incumbent would have to at least match the utility of the outside offer (or its employees would accept the outside offer). To do so, it must reduce the assignment for

[26] If over-assignment was possible, then incumbents could deter these offers using the over-assigned employees.

employees in that pool to below w_2/y; at the same wage but a sufficiently low assignment, their utility would be too high for the outside firm to poach them.

Note that it is not feasible to construct pools that meet this criterion for all $w_2 > 0$. In particular, the outside offer for $w_2 = y$ is $(w_2^*, \beta_2^*) = (y, 1)$; there are no more skilled employees that can be in included in the pool to facilitate a more attractive offer. As a consequence, all employees with $\alpha = 1$ are assigned to the pool with $w_2 = y$ and are assigned $\beta(1) = 1$. At all other wages, they will be assigned at or below w_2/y and thus generate lower profits.

To demonstrate that pools that meet this criterion are not viable in equilibrium, consider the alternatives available to the incumbent for the under-assigned employees included in the pool. Assume we have an equilibrium with a pool with a wage w_2 where the maximal assignment is below $\frac{w_2}{y}$. Define the maximum profits employees in this pool generate as π_{max}. These profits are below those that would be generated if the most skilled employees in this pool were assigned $\frac{w_2}{y}$:

$$\pi_{max} < w_2(1 + s_1) + \left(1 - \frac{w_2}{y}\right) * s_0 - w_2 = w_2 * s_1 + \left(1 - \frac{w_2}{y}\right) * s_0 \qquad [A5]$$

Per the above, this pool must contain more skilled employees who are under-assigned. These employees will all generate π_{max}. Let the most skilled employee in the pool have skill $\alpha' > \frac{w_2}{y}$. This employee can feasibly be assigned to any pool where the maximum assignment is equal to or below her ability. In order for this to be an equilibrium, reassignment to those alternative pools cannot increase incumbent profits. Therefore, the profits π_{max} are weakly greater than the analogous maximum profit in those alternative pools. At the same time, the most skilled members of those other pools can be feasibly reassigned to this pool (which has a lower maximum assignment); therefore, it must be the case that all feasible alternative pools generate the same maximum profits per employee (or they would be reassigned to this pool). This implies that every feasible alternative pool must contain employees that are under-assigned: pools with no under-assigned employees, including those with a null set of employees, allow greater utilization of their employees and thus greater profits.

In fact, this implies that all pools with greater assignments must contain under-assigned employees. If there was a maximum level of skill at which employees were under-assigned, that employee would have feasible alternatives at which she could be more fully utilized: those pools with maximum assignments up to her ability, which allow for full utilization because they do not

have under-assigned employees by assumption. But we know that employees with $\alpha = 1$ are paid a wage of y, assigned 1 of the skilled task, and generate profits of $y * s_1 > \pi_{max}$. In other words, this pool has no under-assigned employees. Therefore, it is not the case that all pools with greater assignments contain under-assigned employees, and there must be feasible alternatives for the under-assigned employees that increase profits. This invalidates the only remaining form of prospective pooling equilibria. QED

Proposition 3: With a salary ban, the unique equilibrium has an inefficient promotion threshold $\alpha' > \beta'$ and a promotion wage $w_p^* > 0$ such that:

i) Employees with $\alpha < \alpha'$ are not promoted; incumbents offer a second period wage-assignment pair of $\big(w_{np}, \beta(\alpha)\big) = (e * \min(\alpha, \beta'), \min(\alpha, \beta'))$, and outside firms offer $(w_{np}^*, \beta_{np}^*) = (0,0)$

ii) Employees with $\alpha \geq \alpha'$ are promoted; incumbents offer a second period wage-assignment pair of $\big(w_p(\alpha), \beta(\alpha)\big) = \big((w_p^* - e * \beta_p^* + e * \alpha,\ \alpha\big)$ and outside firms offer (w_p^*, β_p^*), where $\beta_p^* = \underset{\beta}{\operatorname{argmax}}\, E[Y_{i,2}^* - \beta * e \mid \alpha \geq \alpha']$, the assignment that maximizes the utility of employees with skill above the promotion threshold when employed by the outside firm

No workers change firms, and new entrants to the labor market receive positive wages that reflect the expected lifetime profits from new hires but are not assigned any of the skilled task.

Proof: The reasoning follows that of *Proposition 1* very closely, with small adjustments to account for the cost of effort. Like in *Proposition 1*, outside firms cannot make positive offers to employees who are not promoted. That allows the incumbent to pay them the minimum possible wage. That minimum wage is no longer 0, however, it is $e * \beta$. Similarly, the outside offer to promoted employees is now a wage-assignment pair that delivers utility equal to $w_p^* - e * \beta_p^*$. Incumbent firms must match this utility in order to retain their promoted employees. Therefore, in order to assign these employees their full capacity α, they must pay wages of $w_p^* - e * \beta_p^* + e * \alpha$, which equates utility with the outside offer.

We derive the analog to [A1]:

$$\alpha' = \beta' + (w_p^* - e * \beta_p^*)/K \qquad [A6]$$

where $K = y * (1 + s_1) - s_0 - e > 0$. Similarly, the analog to [A3] is:

$$w_p^* = E[Y_2^* | \beta = \beta_p^*, \alpha \geq \alpha'] \quad \text{[A7]}$$

Like in *Proposition 1*, the equilibrium is defined by the crossing of the functions in [A6] and [A7] in the feasible domain of α' ($\beta' < \alpha' \leq 1$). Evaluating [A5] at $\alpha' = \beta'$ yields $w_p^* = e * \beta_p^*$. This would result in a utility of 0. Yet we know that the minimum utility that the outside firm can offer when $\alpha' = \beta'$ is positive (it must be at least $\beta' * (y - e)$). Therefore, the wage in [A7] must be above $e * \beta_p^*$. As in *Proposition 1*, [A7] yields $w_p^* = y$ when $\alpha' = 1$. Therefore, the functions cross iff [A6] yields a wage of at least y when evaluated at $\alpha' = 1$:

$$w_p^*(\alpha') | \alpha' = 1 \geq y \rightarrow$$

$$(1 - \beta') * K + e \geq y \rightarrow$$

$$\beta' \leq (y * s_1 - s_0)/[y * (1 + s_1) - s_0 - e] \quad \text{[A8]}$$

The denominator in [A8] is greater than that in [A4], implying that the constraint on β' is looser; it is more likely to have some promotions when effort is costly. This is because wages rise with ability even amongst the employees who are not promoted (in order to just compensate for effort) and thus the penalty of needing to pay a wage upon promotion has less of an effect on profits. As in *Proposition 1*, it is possible that there is no feasible solution, in which case no employees are promoted. Regardless, incumbent firms generate positive second period profits, and these are reflected in the first period wages. QED

Proposition 4: Under an equality mandate, the equilibrium wages and assignments of employees who are not promoted are bound above:

i) $w_{np} \in [0, s_0]$

ii) $\beta_{np}(\alpha) \in [0, s_0/y]$

Proof: For Part (i), assume that we have an equilibrium where $w_{np} > s_0$. As this is an equilibrium, the incumbent wage is just high enough to match the utility of the outside offer and retain the employee with the greatest assignment to the skilled task:

$$w_{np} = w_{np}^* - \beta_{np}^* * e + \beta_{np}^{MAX} * e \quad \text{[A9]}$$

If the utility of the outside offer was zero, then [A9] implies that the incumbent wage is equal to $\beta_{np}^{MAX} * e$, and the incumbent would assign each employee up to her ability (i.e., it would not over-assign any employee who was not promoted, since doing so does not induce the employee

to accept the outside offer). However, the outside firm could then make profitable hires by deviating to a more attractive offer. For example, an offer of $\left(w_{np}^{*}, \beta_{np}^{*}\right) = (w_{np}, \beta_{np}^{MAX} - \varepsilon)$ is accepted by all employees with $\beta_{np} \epsilon (\beta_{np}^{MAX} - \varepsilon, \beta_{np}^{MAX}]$, and these hires generate profits of $\beta_{np}^{MAX}(y - e) - \varepsilon * y$. For a small enough choice of ε, these profits are positive. We conclude that the utility of the outside offer must be positive.

The incumbent only retains employees that it offers a utility at least as high as the outside offer. The incumbent will strategically exploit this to maximize its profits. It will assign its lower skilled, unprofitable workers enough of the skilled task that they accept the outside offer and assign its more skilled (and profitable) employees sufficiently little of the skilled task that they are retained. For example, any employee with $\alpha = 0$ generates negative profits for the incumbent because her wage is above her maximum productivity of s_0 by assumption. Therefore, the incumbent would prefer that this employee accept the outside offer. In general, there is a threshold level of skill α^* below which employees generate negative profits when paid a wage of w_{np}:

$$\alpha * y * (1 + s_1) + (1 - \alpha) * s_0 - w_{np} < 0 \rightarrow$$
$$\alpha < (w_{np} - s_0)/[y * (1 + s_1) - s_0] = \alpha^* \qquad \text{[A10]}$$

It is clear from [A10] that when $w_{np} > s_0$, it must be the case that $\alpha^* > 0$, and some employees generate negative profits for the incumbent. The incumbent would assign those employees a sufficiently high $\beta_{np}(\alpha | \alpha < \alpha^*)$ that they accept the outside offer (for example, an assignment of w_{np}/e results in a utility of 0). However, these employees are guaranteed to generate negative profits at the outside firm at any positive wage: they are less skilled than the general population and thus cannot be profitably assigned any of the skilled task. Therefore, the outside firm can increase its profits by deviating from this strategy. For example, an offer of offer of $\beta_{np}^{*} = w_{np}/e$, which provides utility of 0, is not accepted by any employees and thus generates profits of 0. We conclude that we cannot have an equilibrium with $w_{np} > s_0$.

For Part (ii), the upper bound on the assignment follows similar logic to *Proposition 2*: if an employee is paid below her outside marginal product, she can be poached. For any employee assigned more than w_{np}/y, there exists an $\varepsilon > 0$ small enough that an outside offer of $\left(w_{np}^{*}, \beta_{np}^{*}\right) = (w_{np}, \frac{w_{np}}{y} + \varepsilon)$ is accepted by the employee, and generates profits of $\varepsilon * y$. Because $w_{np} \leq s_0$, it is worth retaining all employees, and the incumbent would deviate to a

lower assignment or a higher wage and retain the employee (and her production linked to firm-specific human capital). QED

Proposition 5: Under an equality mandate, the equilibrium assignments made by incumbents to employees who are promoted are bound above and below:

i) $\beta_p(\alpha) \leq \max(\beta', \frac{w_p}{y})$

ii) $\beta_p(\alpha) \geq \max(\beta', (w_p + w_{np} * K)/C]$

where $K = s_1 + s_0/y$ *and* $C = y * (1 + s_1) - s_0$

Proof: The reasoning on the upper bound in Part (i) exactly follows that of the analogous upper bound in *Proposition 4*. Given a wage of w_p, any employee assigned above w_p/y can be profitably poached by the outside firm with an offer of the form $(w_p, \frac{w_p}{y} + \varepsilon)$. The only caveat is that, if the maximum assignment is β', and some employees with skill below β' are promoted, then the outside firm cannot necessarily make an offer as attractive as $y * \beta'$, if the losses from over-assigning some employees are too large.

For Part (ii), we first recognize that a promoted employee must have an assignment greater than β' by definition. However, it is possible that an even tighter constraint applies. An employee who is not promoted but has skill above β', would be assigned $\beta = \frac{w_{np}}{y}$ if not promoted. That results in profits of:

$$\pi_{np} = \beta * y * (1 + s_1) + (1 - \beta) * s_0 - w_{np} = w_{np} * s_1 + \left(1 - \frac{w_{np}}{y}\right) * s_0 \qquad [A11]$$

Only employees who generate at least that level of profits will be promoted. The profits upon promotion are:

$$\pi_p = \beta_p * y * (1 + s_1) + (1 - \beta_p) * s_0 - w_p \qquad [A12]$$

Promotion requires that $\pi_p \geq \pi_{np}$. We solve for the constraint on β_p:

$$\beta_p \geq [w_p + w_{np}\left(s_1 + \frac{s_0}{y}\right)]/[y * (1 + s_1) - s_0] \qquad [A13]$$

QED

To demonstrate that equilibria of the proposed for exists under an equality mandate, we give one particular example.

Under an equality mandate, an equilibrium exists with a promotion threshold $\alpha' > 0$ *such that:*

i) *Employees with* $\alpha < \alpha'$ *are not promoted; incumbents offer a second period wage-assignment pair of* $\left(w_{np}(\alpha), \beta_{np}(\alpha)\right) = (0, 0)$*, and outside firms offer* $(w_{np}^*(0), \beta_{np}^*(0)) = (0,0)$*;*

ii) *Employees with* $\alpha \geq \alpha'$ *are promoted; incumbents offer a second period wage-assignment pair of* $\left(w_p, \beta_p(\alpha)\right) = \left((y - e) + e * \beta_{max},\ \min\left(\max\left(\alpha, \beta'\right), \beta_{max}\right)\right)$ *for some* $\beta_{max} \leq 1$*, and outside firms offer* $(w_p^*, \beta_p^*) = (y, 1)$*;*

iii) *Outside firms make (off-equilibrium) offers to any employee who is not promoted but paid a positive wage equivalent to the offers they make to promoted employees:* $(w_{np}^*(w), \beta_{np}^*(w)) = (y, 1)$ *for all w > 0.*

No workers change firms, and α' *is the skill level at which incumbents are indifferent between not promoting (and assigning none of the skilled task) and promoting.*

Proof: First we note that the outside offer for any employee paid a positive wage generates a utility of *(y-e)*. The incumbent must at least match this utility to retain any employee paid a positive wage. This implies that the promoted employee assigned the highest amount of the skilled task (who has the lowest utility) will have exactly this level of utility. Assume that employee is assigned β_{max}. This level of assignment implies the wage necessary to pay promoted employees:

$$w_p = (y - e) + e * \beta_{max} \qquad \text{[A14]}$$

For example, if the incumbent chooses to assign the most skilled employee to $\beta_{max} = 1$, then it must pay a wage of y. If it settles for a lesser assignment (we will determine the optimal β_{max} below) it can reduce the wage. This wage then determines the optimal promotion threshold α'. It is the level of skill at which the incumbent firm is indifferent about the promotion status of the employee. If not promoted, the employee generates profits of s_0. Since promotion entails paying a wage of w_p, the employees who are worth promoting generate enough production to make it worth paying them that wage. We start by assuming that the promotion threshold is above the public threshold of β'.

$$\pi_p = \pi_{np} \rightarrow$$

$$\alpha' y(1 + s_1) + (1 - \alpha')\, s_0 - w_p = s_0 \rightarrow$$

$$\alpha'[y * (1 + s_1) - s_0] = w_p \rightarrow$$

$$\alpha' = \frac{w_p}{[y*(1+s_1) - s_0]} \qquad \text{[A15]}$$

If [A15] is in fact above β', then this is the optimal promotion threshold. However, if [A15] is below β', then we need to adjust the threshold for over-assignment, since promotion requires an assignment of at least β':

$$\pi_p = \pi_{np} \rightarrow$$

$$\alpha' y(1 + s_1) + (\beta' - \alpha') * L + (1 - \beta')\, s_0 - w_p = s_0 \rightarrow$$

$$\alpha'[y * (1 + s_1) - L] = w_p - \beta'(L - s_0) \rightarrow$$

$$\alpha' = \frac{w_p - \beta'(L - s_0)}{[y*(1+s_1) - L]} \qquad \text{[A16]}$$

Therefore, either [A15] or [A16] define the threshold. If it is [A16], then some employees are over-assigned. Note that both the numerator and denominator of [A16] are positive, and thus $\alpha' > 0$ for any choice of β_{max}.

The incumbent maximizes its profits by using its employees to the fullest extent possible, conditional on two constraints. At most, it can assign β_{max}. At least, it must assign β'. Therefore, for all $\alpha > \alpha'$, we have $\beta_p(\alpha) = (\min(\max(\alpha, \beta'), \beta_{max}))$.

The incumbent will choose the profit-maximizing level of β_{max}, based on these assignments to the skilled task. As it reduces β_{max}, it can reduce the wage it must pay its promoted employees, and thus is able to promote a wider range of employees. The optimal choice will satisfy the following:

$$\beta^*_{max} = \underset{\beta_{max}}{\operatorname{argmax}}\, \pi(\beta_{max}) \rightarrow$$

$$\beta^*_{max} = \underset{\beta_{max}}{\operatorname{argmax}} \int_{\alpha'(\beta_{\max})}^{1} \Big\{ Min(\alpha, \beta_p(\alpha)) * y * s_1 + \max\big(0, \beta_p(\alpha) - \alpha\big) L + (1 - \beta_p(\alpha)) * s_0 - w_p(\beta_{max}) \Big\} f(\alpha) d\alpha \qquad \text{[A17]}$$

The outside firm cannot make greater offers to the employees who are not promoted. Any such offer would be accepted by all those employees, who have lower skill than the general population and thus cannot be assigned any of the skilled task. Similarly, the outside firm cannot raise its offer to promoted employees; it is already offering the highest possible utility, equal to *(y-e)*. Outside firms have no incentive to change the off-equilibrium offer made to employees

who are not promoted but are paid a positive wage, as these offers are not accepted by any employees. A justification for this offer is that the outside firm is itself subject to the equality mandate, and thus should in principle be restricted to only two wages. The off-equilibrium offer to employees who are not promoted preserves its compliance with the equality mandate, since it is the same wage offered to promoted employees. We will see below that a high offer is necessary: it provides the equality mandate version of a steady state. It must provide sufficient utility such that the incumbent has no incentive to raise its wage and better utilize its employees.

Incumbent firms choose the threshold α' optimally, and so are already maximizing profits conditional on assigning none of the skilled task to employees who are not promoted and providing utility of at least *(y-e)* to promoted employees. The equality mandate prevents them from reducing the wage of employees with skill below β^*_{max}, who are overpaid vis-à-vis the outside offer (i.e., they earn utility above that implied by the outside offer, because they are paid the maximum wage but assigned less than β^*_{max} of the skilled task).

The only remaining deviation to consider is that the incumbent may prefer to raise the wage of the unpromoted employees and more fully utilize those with some skill. The off-equilibrium outside offer ensures that doing so requires awarding employees a utility of at least *(y-e)* to retain them. Specifically, if the incumbent wished to assign an employee β of the skilled task, w_{np} must be at least $(y-e)+\beta*e$. The profits assigning a given β are:

$$\pi(\beta) = \beta * y(1+s_1) + (1-\beta)\, s_0 - w_{np} \rightarrow$$
$$\pi(\beta) = \beta * [(y-e) + (y*s_1 - s_0)] + s_0 - (y-e) \qquad [A18]$$

where we substitute the required salary for that assignment. Both $(y-e)$ and $(y*s_1-s_0)$ are positive by assumption, implying that [A18] is increasing in β. Therefore, a sufficient condition for deviations of this kind to be unprofitable is that even the maximum assignment to unpromoted employees of β' is unprofitable. Note that this is more strict than necessary because raising the wage sufficiently high to allow for an assignment of β' entails overpaying many employees (and possibly over-assigning some with low skill such that they accept the outside offer), and the average profits from this type of deviation will be much lower than the profits

from the most skilled employees. That said, we can substitute β' into [A18] and compare to the equilibrium profits of s_0:

$$\beta' * y(1 + s_1) + (1 - \beta')\, s_0 - w_{np} \leq s_0 \rightarrow$$

$$y * s_1 - \ s_0 \leq (y - e) * (1 - \beta')/\beta' \qquad \text{[A19]}$$

We conclude that if the firm specific human capital associated with the skilled task s_1 is not too high, then [A19] is satisfied and the proposed equilibrium is robust to this type of deviation. Although the necessary condition is a looser constraint on s_1, it is the case that a pure strategy equilibrium is not possible if s_1 is too high. The maximum utility the outside firm can ever offer is *(y-e)*. If s_1 is very high relative to s_0, then deviations to a wage above s_0 for unpromoted employees can be impossible to deter, and per *Proposition 4* a wage above s_0 cannot be an equilibrium.

Average wages are lower than is the unrestricted case considered in *Proposition 2* because employees with $\alpha \in (0, \alpha')$ are underutilized. This reduces both the second period wages earned by those employees, and the second period profits from firm-specific human capital, and thus reduces the first period wage as well. If $\beta^*_{max} < 1$ then some promoted employees are also underutilized. QED