The Fact-Law Distinction: Strategic Factfinding and Rulemaking in a Judicial Hierarchy

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Abstract

A fundamental but understudied procedural institution of American law is that appellate courts defer more to trial courts' findings of fact than to their conclusions of law. I formally model this procedural institution, showing how trial courts use factfinding to achieve their preferred outcome and how appellate courts craft rules in anticipation of trial courts' strategic factfinding. Trial courts do not always report facts truthfully. Appellate courts do not commit to consistent rules, but consistent rules may emerge in equilibrium, creating a misleading appearance of judicial commitment to legal consistency. The model shows that preference divergence between trial and appellate courts has a nonmonotonic effect on factfinding. In addition, fact deference can explain suboptimal rulemaking and reversals even when there is no uncertainty about the likelihood of review or the reviewing court's ideal rule. Finally, the model is useful in understanding why the institution of fact deference persists.

1 Introduction

The literature on judicial politics focuses mostly on supreme and appellate courts and their interaction. But the vast majority of judicial activity in the nation takes place at trial courts and their interaction with intermediate appellate courts (e.g., Administrative Office of the U.S. Courts, Federal Judicial Caseload Statistics 2017). The work of trial courts is both distinctive and important. Whereas appellate courts decide discrete issues arising in a case, trial courts manage the whole case, shepherding the parties through litigation from when the complaint is filed to when the remedy or sentence is determined, in the process retaining substantial discretion over many important decisions. As the celebrated trial judge Charles Wyzanski wrote to his Senator when declining a nomination to the First Circuit Court of Appeals (quoted in Murphy, Pritchett and Epstein (2006), 113-114),

> The District Court gives more scope to a judge's initiative and discretion. His width of choice in sentencing defendants is the classic example. But there are many other instances. In civil litigation a District Judge has a chance to help the lawyers frame the issues and develop the facts so that there may be a meaningful and complete record. He may innovate procedures promoting fairness, simplification, economy, and expedition. ... The District Judge so often has the last word. Even where he does not, heed is given to his estimates of credibility, his determination of the facts, his discretion in framing or denying relief upon the facts he found.

This paper is about a chief source of trial judges' discretion—the deferential standard of review of trial courts' findings of fact. When a decision is appealed, trial courts' legal determinations are reviewed *de novo*—literally, "anew," meaning the appellate court decides the issue according to its own best legal interpretation, without deference to the decision below. By contrast, factual determinations are reviewed under the "clear error" standard, meaning the appellate court defers to the trial court's determination and does not overturn it unless clearly incorrect. Reversal is warranted under the clear-error standard only if the reviewing court "is left with the definite and firm conviction that a mistake has been committed" (*Easley v. Cromartie*, 532 U.S. 234, 235 (2001)). Factual determinations are thus reviewed "with a serious thumb on the scale" in favor of the factfinder (*U.S. Bank National Association v. Village at Lakeridge, LLC*, 138 S. Ct. 960, 966 (2018)). Clear-error review of factual determinations, as contrasted with *de novo* review of legal rulings, is required by the Federal Rules of Civil Procedure (Rule 52(a)(6)), as well as federal and state caselaw,[1] and applies in both civil and criminal proceedings.[2] As such, it is a fundamental procedural feature of the American judicial system.

This procedural institution has important real-world implications. Trial judges engage in factfinding in many contexts. They find the ultimate facts in bench trials. More commonly, they find facts in conjunction with making legal determinations. For example, criminal defendants often bring a motion to suppress evidence on the grounds that it was illegally obtained, and in deciding the legal question of whether the evidence should be suppressed the trial court first has to determine the facts of how the police got the evidence (see Section 4). Judge-made factual determinations also pervade legal determinations in patent law (see Section 5), trademarks, antitrust, bankruptcy, contracts, and many other areas.[3]

The justifications commonly given for deferential review of factual judgments are that trial judges are better positioned to assess witness credibility, and that conducting a fresh round of factfinding every time a case is appealed would be immensely costly to the judicial system. (Appellate hearings consist of lawyers' arguments, not presentation of evidence and witness testimony.) Whatever the

[1] See, e.g., *Highmark Inc. v. Allcare Health Mgmt. Sys., Inc.*, 134 S. Ct. 1744, 1748 (2014); *United Computer Sys., Inc. v. AT & T Corp.*, 298 F.3d 756, 760 (9th Cir. 2002); *New Windsor Volunteer Ambulance Corps, Inc. v. Meyers*, 442 F.3d 101, 113 (2d Cir. 2006); *Dual-Temp of Illinois, Inc. v. Hench Control, Inc.*, 821 F.3d 866, 869 (7th Cir. 2016); *People v. Louis*, 728 P.2d 180, 189 (1986) (California); *Guzman v. State*, 955 S.W.2d 85, 87 (1997) (Texas); *In re Cross*, 327 P.3d 660, 673 (2014) (Washington); *Casavant v. Norwegian Cruise Line Ltd.*, 952 N.E.2d 908, 911 (2011) (Massachusetts).

[2] Mixed questions of law and fact are reviewed under the clear-error standard or *de novo* depending on whether factual or legal issues predominate (*Village at Lakeridge*, 138 S.Ct. at 966–67).

[3] Factfinding by trial judges should not be confused with factfinding by juries at trial. Appellate deference to a jury's factfinding is nearly absolute (assuming the case was properly allowed to proceed to trial in the first place). But only a miniscule portion of cases actually go to trial (e.g., about 1.5 percent of civil cases and 9 percent of criminal cases in federal courts, see Administrative Office of the U.S. Courts, U.S. District Courts – Judicial Business 2017). Factfinding by trial judges is much more common; it often occurs regardless of whether the case ultimately proceeds to jury trial or is disposed of otherwise.

merits of these justifications, the more interesting question from the perspective of positive social science is how this procedural institution structures the strategic interaction of trial and appellate courts.

I present a formal model to explore the implications of deferential fact review for factfinding and rulemaking in a judicial hierarchy. The model takes clear-error review seriously by requiring the appellate court to defer to the trial court's factual determinations as long as they fall within certain bounds. This deference gives the trial court, which knows more than the appellate court about the facts, some discretion in factfinding, which it can use strategically to obtain its preferred outcome in a case. If the appellate court believes that the trial court is using its discretion to distort the facts, then it can distort applicable law to compensate. But distorting the law is costly because it establishes a bad guide for future cases and conduct. So the appellate court must probabilistically trade off the present benefits of correctly deciding the case against the future costs of fixing a bad rule. The trial court's strategic factfinding occurs in the shadow of this tradeoff.

The model shows that, given deferential fact review and the trial court's informational advantage, the trial court will not always truthfully report case facts. Nor will the appellate court employ a consistent rulemaking strategy, in the sense of committing to the same rule (e.g., its ideal point); rather, the appellate court threatens rule distortion in response to certain factual determinations by the trial court. Nevertheless, in the pure-strategy equilibrium the rule always ends up at the appellate court's ideal point. The fact that off-path rule distortion is not empirically observable creates the impression of judicial commitment to legal consistency, but the model shows that there will be no such commitment.

The mixed-strategy equilibrium shows how fact discretion can lead to rules that both courts dislike (Pareto-dominated rules). It also shows why the trial court may be reversed even in the absence of uncertainty about the appellate court's ideal rule or the likelihood of review.

The model produces interesting comparative statics. Factfinding increases in the trial court's valuation of a case's importance and in how fact-intensive the case is, and decreases in the appellate court's valuation of case importance and the trial court's cost of factfinding. Less intuitively, preference divergence between the courts has a nonmonotonic effect on factfinding.

Comparing the American judicial system to one without deferential fact review shows that the institution of deference benefits appellate courts if and only if their preferences are closely aligned with trial courts. This suggests that understanding why the institution persists requires looking beyond the familiar story that the principal voluntarily delegates discretion to elicit the agent's greater effort.

2 Relation to Literature

This paper builds on and advances the formal literature on judicial hierarchy (see Kastellec (2017) for an overview). I employ the "case space" framework, described in Section 3.1, which has become standard in the literature (see Kornhauser (1992), Lax (2011)). A prominent virtue of the case space approach is that it distinguishes courts' dispute-settling and rulemaking functions while showing how the functions are linked. My model focuses on a potential tradeoff between these two considerations.

I also adopt from the literature a principal-agent perspective with a focus on informational asymmetries between lower and higher courts. But, unlike most of the literature, this is not an auditing model. The focus on auditing goes back to an influential article by Cameron, Segal and Songer (2000). In that model, some case facts are "public," observable to both higher and lower courts, and some are "private," observable to the lower court but not to the higher court unless it decides to pay the cost of reviewing the case. The heart of the analysis is whether the higher court audits the lower court. A similar theme pervades subsequent literature. For example, Lax (2003) relaxes the assumption that the higher court is a unitary actor to investigate the impact of the "rule of four" on the Supreme Court's certiorari decision; Carrubba and Clark (2012) and Clark and Carrubba (2012) explore judicial utility functions defined not only over dispositions but also over rules and opinion quality; Beim, Hirsch and Kastellec (2014) investigate how "whistleblowing" by a lower-court judge (or a non-judicial actor like the Solicitor General) can help higher courts induce lower-court compliance; Badawi and Baker (2015) investigate why appellate courts spend resources on developing precedent rather than issuing summary orders. Each of these works enriches our understanding of judicial politics by exploring new territory. One feature they have in common, though, is that they are about a principal deciding whether to audit an agent. What makes the wheels turn in each of these models is the cost of review, which the principal can pay and learn the facts, or not pay and be free of the cost but ignorant of the facts.

The auditing metaphor is a good way of capturing the logic of what one might call "second-level" review—that is, a supreme court's review of appellate court decisions (as well as *en banc* review in federal circuit courts). But it is not a good fit for "first-level" review—that is, intermediate appellate court review of trial court decisions—because in that context the higher court is *required* to hear all appeals (appeal "as of right"). My model captures this (more common) setting.

There have been recent efforts to model first-level review (Hübert (2019), Baker and Kornhauser (2017)). Like this paper, these works formally analyze appellate courts' deference to trial courts. But they take "deference" to mean simply affirming a judgment, and there are no standards of review in the models. Lax (2012) also studies strategic rulemaking, including an interesting comparison of rules and standards, but does not model lower courts as strategic actors.[4]

The present work advances the literature by formally analyzing the fact-law distinction in a principal-agent framework. To my knowledge the only formal model of judicial fact discretion is found in Gennaioli and Shleifer (2008).[5] The focus of that paper, however, is welfare analysis of optimal tort damages, not principal-agent relations in the judicial hierarchy. A number of strong assumptions—such that

[4] Other work speaking to trial courts' discretion includes Fischman and Schanzenbach (2012), Huber and Gordon (2007), Schanzenbach and Tiller (2007).

[5] More generally, Tiller and Spiller (1999) discuss the choice between low-cost and high-cost instruments available to an agent. They note that the agent might choose a high-cost instrument (despite its cost) if doing so would make monitoring prohibitive for the principal, and provide a number of illuminating illustrations concerning administrative agencies (pp. 360-362).

there is no limit to trial judges' ability to manipulate facts without fear of reversal, and that there is nothing appellate courts can do about trial courts' factfudging—take out the element of strategic interaction. When the law is settled, analysis of what trial judges will do is reduced to a simple optimization problem (pp. 9, 16). Even when the law is unsettled, Gennaioli and Shleifer assume away much that is of strategic interest by effectively assuming that the appellate court must believe the facts reported by the trial court—the appellate court's utility is calculated as if the reported facts are true (p. 18). These assumptions are appropriate in light of the authors' focus, but my focus on strategic principal-agent interactions demands an entirely different approach. I allow appellate courts' beliefs about true facts to be informed by their knowledge of the possibility of trial judges' factfudging. Moreover, appellate courts can do something about factfudging by making rules that take its possibility into account.

3 Model

3.1 Illustrative Example

The model employs the "case space" framework. Here, a *case* is a point in *fact space* (usually $\mathbb{R}$, but in principle $\mathbb{R}^n$), a *disposition* is a binary measure of the outcome of the case, and a *rule* is a hyperplane dividing the fact space into half spaces corresponding to the two dispositions.[6] To take a simple example, a speed limit of 65 mph assigns the "violation" disposition to cases above 65 and the "no violation" disposition to cases below 65.

I introduce fact discretion to the case space framework, showing how factfinding deference gives trial courts opportunities both to deceive and to help appellate courts. A key driver of strategic interaction in the model is preference divergence between trial and appellate courts. A trial court who disagrees with the appellate court about the law, and who therefore cannot reach its desired outcome under the

[6] It is possible to conceive of more complicated rules, but this paper, like most others, focuses on cutpoint rules.

actual facts of a case, might be able to obtain its preferred outcome by reporting alternative facts. Continuing with the speeding example, suppose the trial court's preferred speed limit is 60 and the appellate court's is 50, and the defendant was driving at 52. The trial court wants to find the defendant not guilty but cannot do so by deciding that the speed limit is 60 (a legal determination); however, the trial court might be able to obtain its preferred disposition, even under the appellate court's preferred speed limit, if it finds that the defendant was driving at 48 (a factual determination).

But the appellate court is not helpless in the face of factfinding. If the appellate court thinks the trial court is fudging, it can nullify the dispositional effect of the trial court's factfinding by changing the speed limit to (just below) 48. But moving the rule is costly for the appellate court, who cares not just about getting this case right but also about setting the right speed limit to govern future cases and conduct. Both elements of this simple example—the trial court's use of fact discretion to obtain its preferred disposition, and the appellate court's tradeoff between disposition and rule utility—are essential in the formal model.

3.2 Setup

A case is decided by a two-level judicial hierarchy (trial and intermediate appellate courts). The appellate court knows the neighborhood of true facts but not their precise location. The trial court, given its closer engagement with the case and sometimes with live witnesses, knows the true facts. The trial court reports some facts to the appellate court. As long as these facts are within the aforementioned neighborhood, they are not "clearly erroneous" and must be taken as the operative facts of the case. The appellate court then chooses a rule, which determines the disposition of the case based on the trial court's reported facts. Each court wants to get the correct disposition in this case (according to its own ideal rule) and to set the rule close to its ideal point (for future, unmodeled, cases).

Formally, a lower court (LC, with ideal point L) and a higher court (HC, with ideal point H, both ideal points being common knowledge) interact as follows:

1. Nature selects the true case facts ($f_t \in \mathbb{R}$) and a signal of case facts ($f \in \mathbb{R}$). LC observes both f_t and f, but HC observes only f. HC knows that true case facts are uniformly distributed on an ϵ-ball around the signal: $(f_t|f = x) \sim U[x - \epsilon, x + \epsilon]\ \forall x$.

2. LC chooses what facts to report, f'. LC may simply report the signal f, a decision represented by $\varphi = 0$, or it may decide at cost c to deviate from the signal, denoted $\varphi = 1$, in which case it can report $f' \in [f - \epsilon, f + \epsilon]$. LC also announces a (provisional) rule r_ℓ, which determines a (provisional) disposition as follows: $d_\ell = \begin{cases} 1 & \text{if } f' < r_\ell \\ 0 & \text{if } f' \geq r_\ell \end{cases}$.

3. HC announces the final rule r, which determines the disposition as follows: $d = \begin{cases} 1 & \text{if } f' < r \\ 0 & \text{if } f' \geq r \end{cases}$.

Payoffs are:

$$U_{LC} = -|r - L| + e_\ell \mathbb{1}(d = d_L) - c\varphi + a\mathbb{1}(r = r_\ell) \tag{1}$$

$$U_{HC} = -|r - H| + e_h \mathbb{1}(d = d_H) \tag{2}$$

where $d_L = \begin{cases} 1 & \text{if } f_t < L \\ 0 & \text{if } f_t \geq L \end{cases}$ and $d_H = \begin{cases} 1 & \text{if } f_t < H \\ 0 & \text{if } f_t \geq H \end{cases}$.

A strategy for LC is the choice of a triplet (r_ℓ, φ, f') given the public signal and the true facts $\big(\sigma_{LC} : \mathbb{R} \times [f - \epsilon, f + \epsilon] \to \mathbb{R} \times \{0, 1\} \times [f - \epsilon, f + \epsilon)\big)$. A strategy for HC is the choice of a rule r given the public signal and LC's factfinding $\big(\sigma_{HC} : \mathbb{R} \times \{0, 1\} \times [f - \epsilon, f + \epsilon] \to \mathbb{R}\big)$. Players are expected-utility maximizers. The solution concept is perfect Bayesian equilibrium. Without loss of generality, assume $L > H$ and $H = 0$.

Here is an explanation of the legal and strategic substance behind the model:

First stage. This stage captures the trial court's information advantage over the appellate court.f can be interpreted as the "paper record" of the case, equally accessible to both courts, which gives an imperfect indication of true case facts (e.g., a police report indicating how fast defendant was driving). ϵ measures how informative the paper record is about the true facts. A larger ϵ denotes a fact-intensive issue, meaning the outcome hinges on specific facts and witness credibility; a smaller ϵ denotes something closer to a pure question of law.[7]

Second stage. This stage captures the trial court's bounded discretion in factfinding. Restricting f' to an ϵ-neighborhood of f gives content to the idea of clear error—beyond a limit, factual determinations become clearly erroneous. Naturally, the parameter (ϵ) that signifies how well the paper record reflects the true facts also delimits how far the trial court may depart from the paper record. I will sometimes refer to the decision to depart from the paper record ($f' \neq f$) as "factfinding" or "costly factfinding." Note that this means a more elaborate factual report, not a more thorough search to learn the facts. In the American legal system, which is adversarial rather than inquisitorial, the trial court's discretion manifests itself in deciding which version of the facts to accept, not in an investigation to uncover facts. (The trial court also announces a rule (r_ℓ), which is required to decide the case at this stage but is not central to the analysis.)

Third stage. The game concludes by the appellate court announcing the rule r, which determines the disposition as per the case space approach (p. 5): Cases below r receive one disposition ($d = 1$), and cases above r get another disposition ($d = 0$). Note that r generates a disposition by reference to case facts *as reported by the trial court* (f'). That the appellate court must accept the trial court's factual report captures the deferential standard of review—as long as the reported facts are within the neighborhood of the public signal where the truth could be, those facts are not reversible.

Payoffs. Both courts' payoffs have a rule component and a disposition component. The disposition component captures each court's desire to get *this* case

[7] The uniform distribution for f_t was chosen for mathematical convenience; any distribution supported on a bounded interval symmetric around f would provide qualitatively similar results. No restrictions are necessary on the distribution of f (beyond being continuous and supported at least on an ϵ-ball around H) for purposes of deriving equilibria. In the limited instances when the direction of comparative statics depended on the distribution of f, I assumed naturally that f is distributed uniformly over a large interval around H.

right; the rule component captures the desire to fix the right rule to govern future cases and future conduct. The rule payoff is a linear loss function of distance between the final rule and a court's ideal rule. (Quadratic loss would work too.)[8] The disposition payoff is e (e_ℓ for LC and e_h for HC), which accrues iff the case's disposition conforms to a court's ideal disposition. Ideal dispositions, denoted d_L and d_H, are the dispositions demanded by each court's ideal rule under the *true* case facts. (HC never learns the truth, so its disposition payoff is in the form of expected utility.) LC's utility function also incorporates a payoff for getting affirmed on appeal (a) and a cost (c) for going beyond the public signal. The cost reflects the extra work the trial court must do to justify reported facts that are different from what the appellate court can directly see.[9] To avoid the trivial equilibrium where there is no factfinding motivated by the desire to flip a case's disposition, I focus on cases where LC's benefit from obtaining its preferred disposition exceeds the cost of factfinding ($e_\ell > c$).

Alternative specifications. The results derived below are robust to certain alternative specifications. In the model LC is restricted to choosing $f' \in B_\epsilon(f)$. A more complicated model, where LC is allowed to set f' anywhere but $f' \notin B_\epsilon(f)$ is clearly erroneous and thus reversible by HC, is easily reducible to the present model because $f' \notin B_\epsilon(f)$ is dominated. In addition, the rulemaking component of LC's strategy (the choice of r_ℓ) and the affirmance payoff (a) are added to capture institutional details, and are not essential to the analysis. The core strategic interaction is LC's factfinding and how HC responds to it.

3.3 Fact Distortion and Varieties of Factfinding

It is useful to begin the analysis by asking whether there are any equilibria in which the trial court always reports the true facts. The answer is no. If LC always tells

[8] The rule payoff can be microfounded by imagining a multiperiod game, with factfinding in every period, where the rule is fixed in the first period and cases are decided in subsequent periods by reference to that rule. Each new legal issue has one such game associated with it.

[9] The cost parameter is not critical to the analysis; its main use is to rule out reported facts that are not meaningfully different from f. Allowing for such babbling would not change the substantively meaningful results.

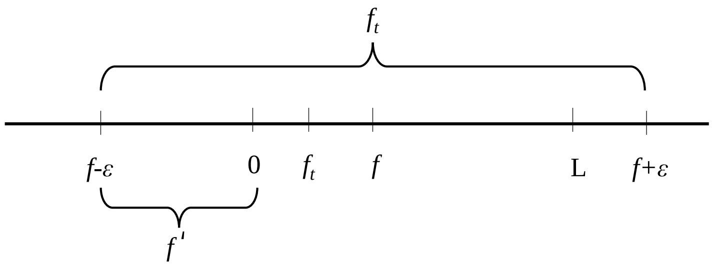


Figure 1: Deceptive factfinding. When HC sets the rule at its ideal point ($r = 0$), setting f' as shown helps LC get its preferred disposition, to the detriment of HC.

the truth then HC always sets the rule at its ideal point. But then, if the two courts' ideal dispositions differ (i.e., $f_t \in [0, L)$) and LC can move the facts from one side of HC's ideal point to the other (i.e., $f \in [0, \epsilon)$), LC has a profitable deviation from truthtelling (Figure 1). This result is formalized in Remark 1.[10]

Remark 1. *There is no equilibrium in which LC always reports the true facts. Formally, there is no equilibrium in which* $\text{sign}\{f'\} = \text{sign}\{f_t\}\ \forall f$.

The remark helps build intuition for why LC might choose to bear the cost of reporting facts other than the public signal. It is useful to distinguish two varieties of factfinding, which I call "helpful" and "deceptive." Helpful factfinding occurs when LC uses its factfinding power to report case facts that are on the same side of HC's ideal point as the true case facts (formally, $\varphi = 1$ and $\text{sign}\{f'\} = \text{sign}\{f_t\}$). Such factfinding can help both LC and HC when their preferred dispositions are the same but the public signal misrepresents the location of true case facts in relation to HC's ideal point, as in Figure 2. (Notice that panels (a) and (b) are possible only if $L < \epsilon$, meaning ideal points are close together.) By contrast, deceptive factfinding (or "factfudging") occurs when LC uses its factfinding power to misrepresent the location of facts with respect to HC's ideal point (formally, $\varphi = 1$ and $\text{sign}\{f'\} \neq \text{sign}\{f_t\}$). Under factfudging, HC's choice of its ideal rule would give LC its preferred disposition to the detriment of HC (Figure 1).

Truthfulness is defined in Remark 1 as $\text{sign}\{f'\} = \text{sign}\{f_t\}$. This is a lenient definition: It requires not literal truthfulness (i.e., $f' = f_t$) but simply an accurate report of whether the facts fall to the left or right of HC's ideal point, which is all that is required for HC to get its ideal disposition under its ideal rule. Unfor-

[10] Proofs omitted from the main text are in the appendix.

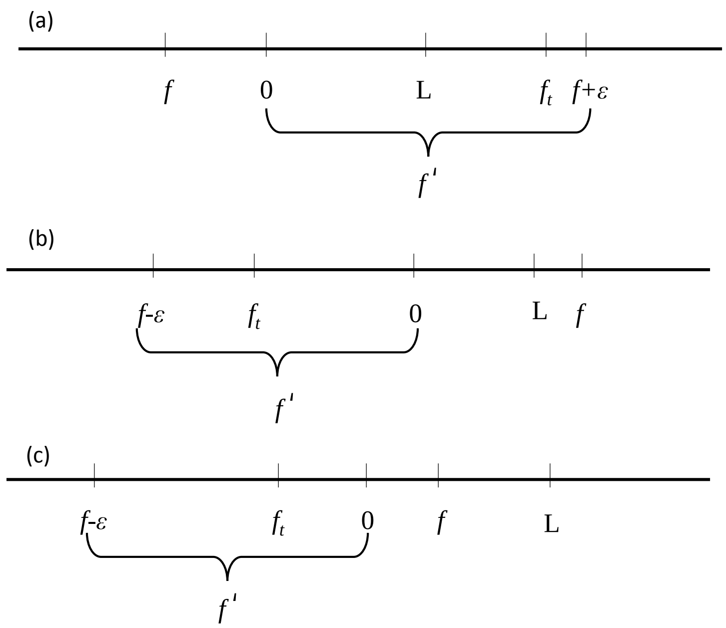


Figure 2: Varieties of helpful factfinding. When HC sets the rule at its ideal point ($r = 0$), setting f' as shown helps both LC and HC get their preferred dispositions.

tunately for HC, Remark 1 rules out truthful equilibria even under this lenient definition. (*A fortiori*, literal truthfulness will also not occur in equilibrium.)

Building on this intuition, it is useful to ask when LC does *not* have an incentive to report facts beyond the public signal. The answer is: for very easy cases. If the public signal is sufficiently far from HC's ideal point ($f \leq -\epsilon$ or $f \geq \epsilon$), then HC knows on which side of its ideal point the true case facts fall, and LC's fact report cannot move the facts from one side of HC's ideal point to the other. So HC would obtain is preferred disposition by choosing its ideal rule, which also uniquely maximizes its rule utility. Therefore, $r = 0$ must be chosen in equilibrium—regardless of the facts reported by LC. Factfinding in such a situation would be pure cost for LC, with no prospect of changing the ultimate rule or disposition, so it would not occur. The result is formalized as follows.

Remark 2. *In very easy cases, LC will not report facts other than the public signal. Formally, if $f \geq \epsilon$ or $f \leq -\epsilon$ then $\varphi = 0$ in equilibrium.*

3.4 Rule Distortion

Next consider whether HC would always set the rule at its ideal point. This would be HC's strictly dominant strategy in a game without fact discretion. Even with fact discretion, considering that LC receives no deference in rulemaking, one might expect that $r = 0$ can be HC's equilibrium strategy. This expectation turns out to be wrong.

To see the intuition behind this result, begin by considering LC's best response to HC's strategy of $r = 0$. For extreme values of f (namely, $f \leq -\epsilon$ or $f \geq \epsilon$), we know by Remark 2 that LC would not report facts other than the public signal. For other values of f, LC might engage in helpful or deceptive factfinding, depending on f_t. If $f \geq 0$ and $f_t < 0$ then LC would set $f' < 0$, helping both courts achieve their ideal disposition (Figure 2(b)-(c)). Likewise, if $f < 0$ and $f_t \geq L$ then LC would benefit both courts by setting $f' \geq 0$ (Figure 2(a)). Deceptive factfinding is also possible: If $f \in [0, \epsilon)$ and $f_t \in [0, L)$ then LC would guarantee its preferred disposition, to the detriment of HC, by setting $f' < 0$ (Figure 1). (Recall that we are holding HC's strategy fixed at $r = 0$.) Following this logic, LC's best response to HC's strategy of $r = 0$ is set forth in Lemma 1.

Lemma 1. *LC's best response to HC's strategy of $r = 0$ is given by $r_\ell = 0$ and the following choices of φ and f'.*

1. *If $f < 0$ then* $\begin{cases} \varphi = 0 & if \;\; f_t < L \\ \varphi = 1 \text{ and } f' \in [0, f + \epsilon] & if \;\; f_t \geq L \end{cases}.$
2. *If $f \geq \epsilon$ then $\varphi = 0$.*
3. *If $f \in [0, \epsilon)$ then* $\begin{cases} \varphi = 0 & if \;\; f_t \geq L \\ \varphi = 1 \text{ and } f' \in [f - \epsilon, 0) & if \;\; f_t < L \end{cases}.$

To ascertain whether $r = 0$ can be HC's equilibrium strategy, one must ask whether $r = 0$ is HC's best response to LC's best-response strategy identified in Lemma 1. The answer is no, and the key to the answer is the region $f \in [0, \epsilon)$. By Lemma 1, in this region LC does not engage in factfinding when $f_t \geq L$, and sets $f' < 0$ when $f_t < L$. For HC to stick to $r = 0$ in the face of $f' < 0$, HC must believe that the factfinding is probably helpful (i.e., the posterior probability that $f_t < 0$ is high), or it must be that changing the rule to counteract LC's likely

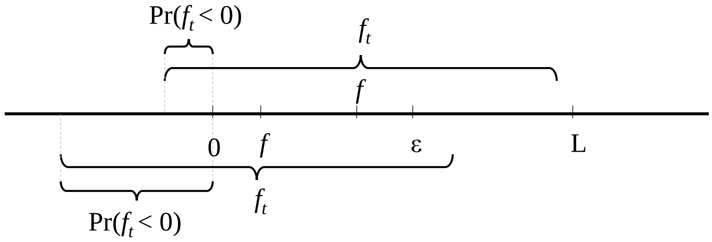


Figure 3: HC's calculus of whether to keep the rule at its ideal point ($r = 0$) when $f \in (0, \epsilon)$ and $f' < 0$. When f is close to ϵ (top of the figure), the probability that the factfinding is helpful is low and countering it by changing the rule would be inexpensive; when f is close to 0 (bottom), the probability of helpful factfinding is higher and changing the rule to counter it could be expensive.

factfudging is too costly (i.e., f' is far to the left of HC's ideal point of 0). Both of these conditions can be satisfied when f is close to 0 (bottom portion of Figure 3). But they cannot be satisfied when f is close to ϵ (top portion of Figure 3)—because then the posterior probability that $f_t < 0$ given $f' < 0$ is low, and because f is so far to the right of 0 that LC cannot set f' sufficiently far to the left of 0 to make a change of rule prohibitively expensive for HC. (Note that the limited nature of LC's ability to manipulate facts, which is bounded by ϵ to give analytical content to the legal concept of clear error, is doing real work here.) This logic leads to Proposition 1.

Proposition 1. *There is no equilibrium in which HC's strategy is $r = 0$.*

This result does not depend on how close the courts' preferences are (parameterized by L) or how much HC cares about case disposition (e_h). The values of L and e_h are indeed relevant—the lower they are, the smaller the interval of f values that would trigger a profitable deviation from $r = 0$—but there is no positive value of L or e_h that can eliminate profitable deviations altogether. A no-rule-distortion-strategy equilibrium can exist only in the limit where the two courts' preferences are identical ($L = 0$) or HC cares nothing about the case ($e_h = 0$).

Given that HC's strategy of consistently setting the rule at its ideal point cannot be supported in equilibrium, a natural followup question is whether *any* consistent rulemaking strategy is supportable. The answer is clearly no.

Corollary 1. *There is no equilibrium in which HC's strategy is $r = \hat{r}$ for some $\hat{r} \in \mathbb{R}$.*

Proof. By the proof of Remark 2, $r = 0$ is HC's unique optimal strategy when $f \leq -\epsilon$ or $f \geq \epsilon$, and by Proposition 1, $r = 0$ cannot be HC's equilibrium strategy. $\square$

It is worth pausing to clarify what has and has not been shown. I have shown that, contrary to plausible intuition, $r = 0$ cannot be sustained as an equilibrium *strategy*. I have *not* shown whether and under what conditions $r = 0$ can be sustained as an equilibrium *outcome*. Even if HC does not follow a strategy of always setting the rule at its ideal point, it's possible that in equilibrium the final rule will always end up there (i.e., $r \neq 0$ occurs off the equilibrium path). That is a possibility to explore when solving for equilibrium.

3.5 Pure-Strategy Equilibrium

This section discusses a pure-strategy equilibrium of the game (see Proposition 2 in the appendix for full characterization and proof). First consider extreme values of f, meaning $f \geq \epsilon$ or $f \leq -\epsilon$. By Remark 2 and Lemma 1 we know that, in these regions, LC does not engage in factfinding and HC sets the rule at its ideal point. Next consider the critical region $f \in [0, \epsilon)$.

In this region, the discussion of rule distortion has shown that for f close to ϵ, setting $r = 0$ cannot be a best response to $f' < 0$. How, then, would HC respond to $f' < 0$? The key to answering this question is to recognize that all rule choices but two are dominated. All choices of $r > f'$ are dominated by $r = 0$ because they all lead to the same disposition and $r = 0$ leads to a strictly higher rule utility. Likewise, all choices of $r < f'$ are dominated by $r = f'$ because they all lead to the same disposition and $r = f'$ provides the strictly highest rule utility. HC's real choice is thus between $r = 0$ and $r = f'$.

Two considerations determine the choice between these two rules: (1) HC's posterior belief that f' is misrepresenting the truth (so choosing $r = 0$ would lead to the wrong disposition), (2) the amount of rule utility that HC would sacrifice by setting $r = f'$ to guard against the probable loss of dispositional utility. For values of f close to 0 (the bottom portion of Figure 3), both considerations lead

HC toward choosing 0 over f'. For values of f close to ϵ (the top portion of Figure 3), by contrast, both considerations pull HC toward choosing f'. These dynamics lead to an equilibrium with a threshold structure. The threshold f^* in the interval $(0, \epsilon)$ specifies the value of f at which, provided LC sets f' as far to the left of 0 as possible (which is in its interest to do), HC's expected utilities from $r = 0$ and $r = f'$ are equal. HC would "tolerate" LC's factfinding below f^* but not above f^*—meaning that if $f > f^*$ and $f' < 0$ then HC would set $r = f'$ to counteract the factfinding.

LC's factfinding is in turn based on HC's anticipated response. When $f > f^*$, LC does not set $f' < 0$ because such factfinding would trigger a response of $r = f'$, which would both nullify the dispositional effect of LC's factfinding and set a rule far from LC's ideal point. When $f \leq f^*$, by contrast, LC can get away with setting $f' < 0$ (provided it sets f' sufficiently far to the left of 0, for example $f' = f - \epsilon$). But setting $f' < 0$ would make sense for LC only if the truth is to the left of its ideal point ($f_t < L$), because otherwise LC can get its preferred disposition under the public signal and without bearing the cost of factfinding. So LC engages in factfinding and sets $f' < 0$ whenever $f \leq f^*$ and $f_t < L$, and does not engage in factfinding otherwise.

Finally, consider the region $f \in (-\epsilon, 0)$. If ideal points are far apart ($L \geq \epsilon$), then by Lemma 1.1 it is a best-response pair for LC not to engage in factfinding and HC to set $r = 0$. (Given $L \geq \epsilon$, the fact that $f < 0$ implies that $f_t < L$, so LC can obtain its preferred disposition without having to bear the cost of factfinding.) If ideal points are close ($L < \epsilon$), however, LC might have an incentive to engage in factfinding because the public signal might misrepresent the true location of case facts with respect to L. When that is the case (i.e., when $f_t \geq L$), LC would like to report $f' \geq 0$ to flip the case's disposition (recall Figure 2(a)). Such factfinding would be not only in LC's interest but in HC's interest as well, because $f_t \geq L$ implies $f_t \geq 0$. So all factfinding in the region $(-\epsilon, 0)$ is helpful, and HC always tolerates it.

To summarize, the structure of pure-strategy equilibrium is as follows: When

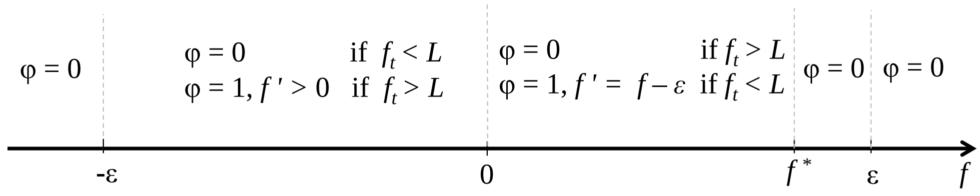


Figure 4: Factfinding in the pure-strategy equilibrium.

$f \leq -\epsilon$ or $f \geq \epsilon$, there is no factfinding and HC sets $r = 0$. When $f \in (-\epsilon, 0)$, HC sets $r = 0$; LC does not engage in factfinding if $f_t < L$ but engages in factfinding and sets $f' \geq 0$ if $f_t \geq L$. When $f \in [0, \epsilon)$, LC does not have an incentive to engage in factfinding when $f_t \geq L$ but does have an incentive to do so when $f_t < L$. HC would tolerate factfinding for $f \leq f^*$ but would not tolerate it for $f > f^*$. Seeing this, LC does not engage in factfinding when $f > f^*$ but engages in factfinding and sets $f' < 0$ when $f \leq f^*$ and $f_t < L$. The equilibrium factfinding outcome is shown in Figure 4.[11]

Note well that, in equilibrium, the rule is always set at HC's ideal point even though HC's rulemaking strategy anticipates setting another rule if factfinding occurs beyond a certain threshold. The equilibrium *outcome* is always $r = 0$ even though $r = 0$ is not HC's rulemaking *strategy*.

3.6 Comparative Statics

First consider how the factfinding threshold f^* moves with different parameters. The threshold increases with fact-intensiveness of the issue ($\partial f^*/\partial\epsilon > 0$). When the case's outcome depends heavily on facts that are better observed by the trial court, and the trial court's factfinding discretion is concomitantly large, the region where the appellate court would tolerate the trial court's factfinding is wide.[12]

By contrast, the factfinding region contracts as HC attaches more importance to case disposition ($\partial f^*/\partial e_h < 0$). When HC cares more about the case, it becomes more willing to forego rule utility to guard against the possibility of

[11] The fact that perfect Bayesian equilibrium places no restrictions on off-path beliefs often results in multiple equilibria in signaling games. However, given reasonable assumptions about HC's beliefs, this equilibrium is the essentially unique pure-strategy equilibrium of the game. Technical discussion of uniqueness is left for a separate appendix.

[12] Thinking back to HC's choice between $r = 0$ and $r = f'$, a larger ϵ makes $r = 0$ more attractive both because it increases the probability that $f_t < 0$ and because it allows LC to increase the amount of rule utility that HC would have to forego to counteract LC's factfinding.

being duped into the wrong disposition. So HC becomes more willing to punish factfinding, which leads LC to do less of it.

The impact of the courts' preference divergence on the factfinding threshold is more nuanced. When the courts' preferences are not terribly distant, preference divergence reduces the factfinding threshold ($\partial f^*/\partial L < 0$). Once preference divergence passes a certain point, however, further divergence ceases to affect the factfinding threshold, which stabilizes at a certain value ($\partial f^*/\partial L = 0$).[13] Figure 5 shows how the factfinding threshold changes with preference divergence, reflecting this piecewise structure.

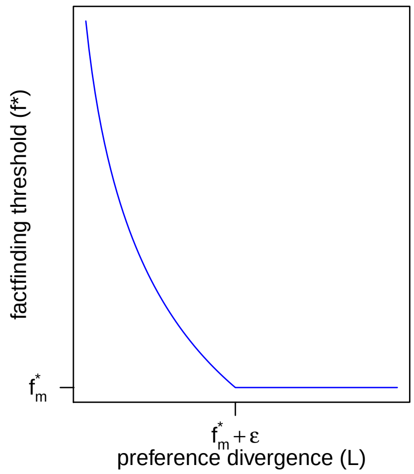
Now consider how probabilities of deceptive and helpful factfinding change with the model's parameters (see appendix for derivations). The effects of LC's dispositional utility (e_ℓ) and factfinding cost (c) are straightforward. Factfinding can occur only if LC cares enough about the case that changing the disposition would be worth the cost of factfinding (i.e., if $e_\ell > c$), so the probability of factfinding decreases in its cost and increases in LC's dispositional utility.

By contrast, both deceptive and helpful factfinding decrease in HC's dispositional utility (e_h). As HC attaches more importance to a case, it becomes more willing to sacrifice rule utility to guard against the possibility of getting the wrong

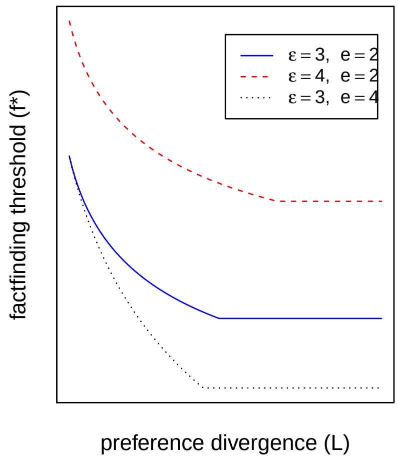
[13] The closed-form expression for the factfinding threshold is:

$$f^* = \begin{cases} \dfrac{2\epsilon + L + e_h - \sqrt{L^2 + e_h^2 + 6Le_h}}{2} & \text{if } L < f_m^* + \epsilon \\ f_m^* & \text{if } L \geq f_m^* + \epsilon \end{cases} \qquad \text{where } f_m^* = \frac{\epsilon^2}{e_h + \epsilon}. \tag{3}$$

The intuition behind the piecewise structure of f^* is as follows (see the proof of Proposition 2 for a more precise treatment). Begin with really far off values of L (i.e., $L \geq 2\epsilon$), where LC would like to set $f' < 0$ whenever $f \in [0, \epsilon)$ (because $f \in [0, \epsilon) \implies f_t < L$). For such a large preference gap, given LC's factfinding incentives, if HC observes factfinding at $f' < 0$ then its posterior belief about the truth is no different from its prior, which is to say $f_t \sim U[f - \epsilon, f + \epsilon]$. Because the posterior does not depend on L, further increases in preference divergence have no effect on the probability that LC's factfinding is deceptive, and a factfinding threshold that is independent of L can be calculated. This threshold, denoted above by f_m^*, demarcates the minimal region of factfinding that HC would tolerate. For lower values of L, HC's posterior belief about the truth conditional on $f' < 0$ is given by $f_t \sim U[f - \epsilon, \min\{L, f + \epsilon\}]$. Here, factfinding might be tolerated above the minimal threshold, and the factfinding threshold might depend on L because the distribution of f_t depends on L. Not so for values of L above $f_m^* + \epsilon$, however, because if HC's expected payoffs from setting $r = 0$ and $r = f'$ are equal when $f_t \sim U[f_m^* - \epsilon, f_m^* + \epsilon]$ then the expected payoff from setting $r = 0$ is lower than from $r = f'$ when $f_t \sim U[f - \epsilon, L]$ for all $L > f_m^* + \epsilon, f > f_m^*$. The upshot is that the threshold f^* is a function of L for $L < f_m^* + \epsilon$ but is constant with respect to L for $L > f_m^* + \epsilon$. At $L = f_m^* + \epsilon$, the value of the function equals the constant.



(a) for fixed ϵ, e_h



(b) for different values of ϵ, e_h

Figure 5: Factfinding threshold as function of preference divergence. Panel (a) plots the function for fixed levels of fact-intensiveness and case importance. Panel (b) shows the same function for three different sets of values for fact-intensiveness and case importance: The blue (solid) line fixes a pair of (ϵ, e_h) as a baseline. The red (broken) line has the same e_h value but a higher ϵ, showing that the factfinding threshold is greater everywhere when the issue is more fact-intensive. The black (dotted) line fixes the same ϵ value as the baseline case but with a higher e_h, showing that HC tolerates less factfinding when it cares more about the case.

disposition. So the threshold (f^*) beyond which HC would not tolerate factfinding moves leftward, and the factfinding region shrinks (Figure 5b).

Factfinding increases in fact-intensiveness (ϵ). Unlike e_h, the impact of ϵ on factfinding does not only come through f^* (recall that ϵ both parameterizes the distribution of true case facts conditional on the public signal and measures LC's factfinding ability), so comparative-static calculations are more involved. In the end, though, the impact of ϵ is monotonic at all levels of preference divergence: Both deceptive and helpful factfinding increase when the case is heavily fact-dependent and LC has greater factfinding discretion.

The impact of the two courts' preference divergence (L) is nuanced and counterintuitive. At first blush one would expect preference divergence to increase factfinding because it expands the conflict region where the two courts' ideal dispositions differ. But in fact the effect of preference divergence depends both on the variety of factfinding (helpful or deceptive) and on how far apart ideal points already are, as shown in Figure 6. When ideal points are close or moderately far apart ($L < L_2$ in Figure 6), an increase in L decreases helpful factfinding

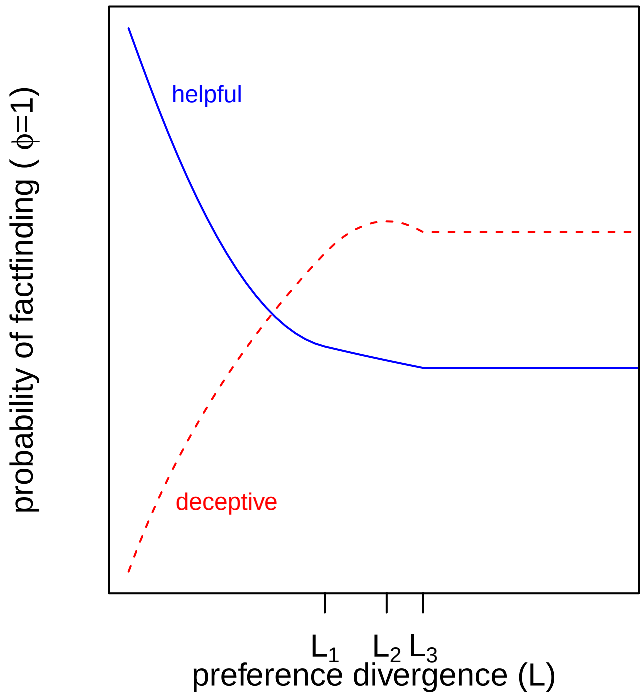


Figure 6: Probabilities of helpful and deceptive factfinding as a function of preference divergence. (Parameter values: $e_h = 2, \epsilon = 2$.) Helpful factfinding is decreasing in preference divergence until it stabilizes for $L > L_3$; by contrast, deceptive factfinding initially increases for $L < L_2$, then decreases for $L \in (L_2, L_3)$, and finally stabilizes for $L > L_3$. Both probabilities are continuous everywhere in L but take three different functional forms for $L < L_1$, $L \in (L_1, L_3)$, and $L > L_3$. Note that $L_1 = \epsilon$ and $L_3 = f_m^* + \epsilon$.

but increases deceptive factfinding. Once preference divergence passes a threshold ($L \in (L_2, L_3)$), further increases in L decrease both helpful and deceptive factfinding. But this effect persists only up to a point; once L passes another threshold ($L > L_3$), further increases in preference divergence cease to have any effect on any kind of factfinding. This second threshold, of course, is the same one beyond which f^* ceases to depend on L (i.e., $L_3 = f_m^* + \epsilon$).

The source of the nonmonotic effect on deceptive factfinding is that preference divergence affects not only whether LC would like to engage in factfinding but also how HC would respond to LC's factfinding, and these effects cut in opposite directions. Preference divergence creates additional incentives for factfinding by expanding the conflict region; but HC knows this, so it becomes less tolerant of factfinding as preferences diverge, which in turn makes LC less inclined to engage in factfinding for fear of being punished by a bad rule. When preferences are close or moderately far apart, the added factfinding incentives created by a marginal increase in preference divergence outweigh the chilling effects of anticipating a harsh response, so preference divergence increases deceptive factfinding. But when preferences are farther apart, the deterrent effect dominates and further preference divergence reduces deceptive factfinding.

3.7 Mixed-Strategy Equilibrium

One feature of the pure-strategy equilibrium is that there is no reversal. But reversals do occur in reality, albeit at a low rate.[14] The mixed-strategy equilibrium presented in this section shows that the same qualitative results can be obtained with reversals in equilibrium.

Proposition 3 in the appendix presents a mixed-strategy equilibrium for when the two courts' ideal points are very far apart ($L \geq 2\epsilon$), analogous to the pure-strategy equilibrium with distant preferences.[15] As before, the players' strategies for $f \geq \epsilon$ or $f < 0$ are straightforward—HC always sets the rule at its ideal point and LC never engages in factfinding. In the middle region ($f \in [0, \epsilon)$), there is a threshold (f^*) below which LC always engages in factfinding and HC tolerates it.

What's different in the mixed-strategy equilibrium is what happens when $f \in (f^*, \epsilon)$. Here, a strategy of always engaging in factfinding by LC is not sustainable in equilibrium because HC would counter LC's factfinding by moving the rule to the left. Seeing this, LC did not engage in factfinding at all in the pure-strategy equilibrium. In the mixed-strategy equilibrium, by contrast, LC sometimes engages in factfinding and HC sometimes tolerates it. The key to sustaining such a strategy profile in equilibrium is that LC's propensity to engage in factfinding depends on the true facts: LC's probability of factfinding is higher when the true facts all below HC's ideal point (helpful factfinding) than when they fall above HC's ideal point (deceptive factfinding).[16] Otherwise, by

[14] In the year ending December 31, 2016, reversals constituted 8.5 percent of decisions in cases terminated on the merits in the U.S. Courts of Appeals. See the Statistical Tables section of the U.S. Courts website (Table B-5).

[15] That is, Case 1 in the proof of Proposition 2. Mixed-strategy analogues to the pure-strategy equilibrium when $L \in [\epsilon, 2\epsilon)$ or $L < \epsilon$ can similarly be characterized.

[16] What is pinned down is the relationship between the probabilities of helpful and deceptive factfinding (π_1 and π_2, respectively), not their absolute values. The relationship is $\pi_2 = \left(\frac{\epsilon - f}{\epsilon + f}\right)\left(\frac{\epsilon - f + e_h}{f - \epsilon + e_h}\right)\pi_1$, which satisfies $\pi_1 > \pi_2$ when $f \in (f^*, \epsilon)$.

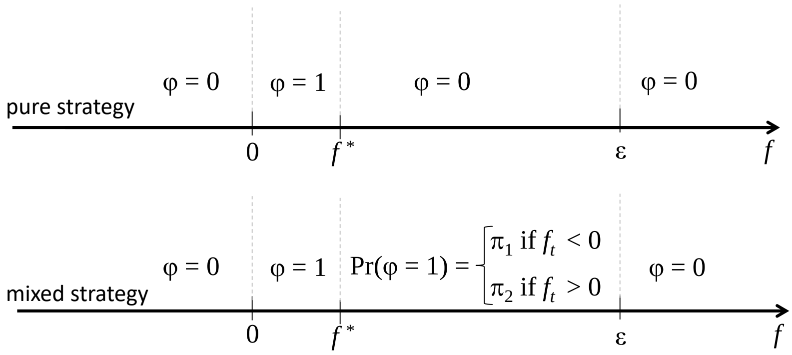


Figure 7: Factfinding in pure- and mixed-strategy equilibria when $L \geq 2\epsilon$.

the same logic as before, HC would never tolerate the factfinding. As it is, HC sometimes tolerates the factfinding ($r = 0$) and sometimes not ($r = f'$), and the latter is what produces reversals. A comparison of factfinding in the pure- and mixed-strategy equilibria appears in Figure 7.

4 Discussion

The first result to highlight is that there is no truthful equilibrium (Remark 1). Though this is not a cheap-talk game, the result resembles the seminal result in Crawford and Sobel (1982) that perfect information transmission is not possible unless the sender and receiver have identical preferences. Here the result is analytically simple but substantively important.

Consider an example that resonates with contemporary concerns surrounding police accountability. During the 1960s, the Warren Court greatly expanded the rights of criminal defendants. One landmark of this "criminal procedure revolution" was *Mapp v. Ohio*, 367 U.S. 643 (1961), which held that evidence obtained by searches and seizures conducted in violation of the Fourth Amendment is not admissible in state criminal prosecutions. Shortly after *Mapp* was decided, a curious change occurred in reported fact patterns in states where such evidence had previously been admissible. As Irving Younger, a former prosecutor and judge, has described, police stopped testifying that they had recovered incriminating evidence after searching the defendant "for little or no reason," and started testifying instead that the defendant panicked upon seeing the police and dropped the evidence on the ground (Younger (1968)). This sudden proliferation of sloppy defendants is so well-known that it was given its own name—"dropsy" testimony

(Barlow (1968), Note (1968)). It is part of a larger pattern of suspicious police testimony, dubbed "testilying," which has caused concern for many scholars, lawyers, and journalists (e.g., Cloud (1994), Mollen et al. (1994), Orfield (1992), Slobogin (1996)).[17]

But these concerned accounts make no reference to deferential fact review, and they portray trial judges simply as being tricked by police perjury. This characterization is probably accurate in many cases—trial judges often cannot detect perjury in a given case even if they suspect a broader pattern of wrongdoing. Nevertheless, the model gives us reason to think of fact deference as part of the institutional structure that contributes to phenomena like testilying. It encourages us to think as trial judges not simply as duped by police perjury (which they may be in many cases) but also potentially as active participants in fact distortion. The result that there is no truthful equilibrium may be interpreted as suggesting that trial judges sometimes credit dubious police testimony even if they can detect it as untruthful.[18] For example, in "dropsy" cases, it is plausible to think that some trial judges hostile to the Warren Court's (rather radical) doctrinal innovations used their factfinding discretion to render the new rules ineffective by crediting questionable police testimony.[19]

The next important result is that there is no equilibrium with a consistent rulemaking strategy (Proposition 1 and Corollary 1). This is important because a consistent strategy seems to respect rule-of-law ideals. The idea that the same law should apply to everyone is deeply embedded in understandings of justice and equality before the law. The fact that appellate courts will not commit to any law

[17] See also Joseph Goldstein, "'Testilying' by Police: A Stubborn Problem," *N.Y. Times*, Mar. 18, 2018; David Kocieniewski, "New York Pays a High Price for Police Lies," *N.Y. Times*, Jan. 5, 1997; Anthony Flint, "Bratton Calls 'Testilying' by Police a Real Concern," *Boston Globe*, Nov. 15, 1995.

[18] This result in no way depends on ascribing sinister or ideological motives to trial judges. The courts' disagreement over rules ($H \neq L$) which gives rise to disagreement over outcomes ($d_L \neq d_H$) may be entirely the product of genuine legal considerations.

[19] Of course, the extent of the problem is an empirical question. Whether trial judges fudged facts cannot be directly empirically verified (which is one reason doing theoretical work is useful), but the theory has empirically falsifiable predictions that, if verified, would increase one's confidence in using the theoretical framework to empirically estimate other components (such as the extent of factfudging) that are not capable of atheoretical empirical verification. See the Conclusion for more thoughts on empirical followups.

appears to deviate from this ideal.

But interpretation of this result is complicated by the fact that, in the pure-strategy equilibrium, the rule is always set at the higher court's ideal point. That is not because the higher court commits to its ideal rule; rather, its strategy threatens deviation from its ideal rule if it observes certain kinds of factfinding given certain public signals. This credible threat, however, deters the lower court from those kinds of factfinding, so there is no need to carry out the threat. The upshot is that the rulemaking *outcome* is consistent, but such consistency is achievable only by a *strategy* that contemplates rule distortion.

It is not obvious what normative conclusion to draw from this result. What is clear is that the distinction between equilibrium strategy and outcome would not be empirically observable or conceptually clear without a formal model. Because rule distortion occurs off the equilibrium path, an observer not engaged with the theory would be misled into thinking that the courts are committed to consistently applying the same rule. The empirical fact of rule consistency paints a picture of legal consistency that masks a more complicated strategic reality.

The mixed-strategy equilibrium preserves the salient qualitative features and comparative statics of the pure-strategy equilibrium while allowing for reversals. With mixed strategies, rule distortion occurs not just off the equilibrium path but on path as well, producing reversals of the lower court. Unlike most judicial-politics models, reversals in this model are not due to the lower court's thinking that its decision might not get reviewed (recall that appeal is as of right). Nor is reversal attributable to the trial court's uncertainty about what rule the appellate court wants (for example because the legal issue is difficult). Rather, reversal occurs because of the structure of fact discretion: Given the appellate court's posterior probability that the lower court's factfinding is deceptive, and given the rule utility it would have to forego to counter potentially deceptive factfinding, the appellate court is indifferent and mixes between tolerating and not tolerating the trial court's factfinding. The choice to not tolerate produces reversals.[20]

[20] The claim here is not that fact discretion is the main reason for reversals. Trial courts' uncertainty about the applicable rule (a mistaken assessment of H) probably explains a greater

When rule distortion occurs ($r \neq 0$), the equilibrium rule is Pareto-dominated for the two courts—there are rules that *both* courts would prefer to the equilibrium rule. (When reversal occurs, $r = f' < 0$, so there are rules (e.g., $r = 0$) that both courts would prefer.) Pareto-dominance occurs because, to deter deceptive factfinding, HC must set the rule to the *left* of its ideal point. Substantively, these results show why we might observe reversals and seemingly suboptimal rulemaking even when the legal issue is not difficult and appellate review is virtually certain.

The most interesting of the comparative statics is on preference divergence. One's intuition might be that preference divergence should increase factfinding because it expands the region where the two courts' ideal dispositions conflict. But the effect is actually nonmonotonic and nuanced. The nuance is traceable to two sources. First, the trial court's factfinding can help as well as hurt the appellate court, and preference divergence has different effects on these different kinds of factfinding. Second, though preference divergence enlarges the conflict region and hence the trial court's incentives for deceptive factfinding, it also heightens the appellate court's suspicion that any factfinding it observes is deceptive rather than helpful, which makes it more willing to punish factfinding, which in turn deters the trial court. The incentive and deterrence effects pull in opposite directions, producing a nonmonotic effect on deceptive factfinding.

5 Fact Deference as Equilibrium Institution

The main purpose of this work is to understand the consequences of fact deference—that is, to take the institution as given and explore its strategic implications. But one might also ask why fact deference persists as an equilibrium institution. A full answer to that question demands a separate inquiry, but my analysis sheds some light on the issue.

One evident explanation, in light of common principal-agent intuitions, is that appellate courts grant discretion to trial courts to benefit from their increased

share of reversals. But the point is that—even in the absence of this more obvious explanation—reversals could still occur because of deferential fact review.

effort in factfinding (e.g., Aghion and Tirole (1997)). To assess the validity of this explanation, one must determine whether trial and appellate courts are better off under the American system or one in which there is no factfinding discretion. So I compare the trial and appellate courts' equilibrium utilities in the game solved above to utilities in a modified setup in which the trial court must report the same facts as the public signal (i.e., $f' = f$). In the modified setup, the public signal can still misrepresent the truth (i.e., $f \neq f_t$), but the trial court cannot report facts other than f—there is no opportunity for either deceptive or helpful factfinding. The results of the comparison appear in Proposition 4.

Proposition 4.

1. *LC is better off with factfinding discretion than without.*
2. *HC is better off with factfinding discretion than without iff the two courts' ideal points are close. Formally, $\exists\, \tilde{L} \in (0, \epsilon)$ such that HC is better off with factfinding discretion than without iff $L < \tilde{L}$.*

The trial court is better off with fact discretion than without, regardless of the level of preference divergence between the two courts. The reason is straightforward: The American system provides an additional factfinding tool to the trial court, which it uses when helpful and declines to use when unhelpful, so it always benefits from the institution. (A trial court could use this added ability in ways that would backfire and hurt it—by engaging in factfinding that is countered by the appellate court's imposition of a punishing rule—but such counterproductive use does not occur in equilibrium.)

By contrast, the appellate court is better off with factfinding than without if and only if the two courts' ideal points are particularly close. The logic of this result goes back to the discussion about different kinds of factfinding. Recall that incentives for helpful factfinding expand when ideal points move close together (Figure 2) and incentives for deceptive factfinding expand when ideal points move away (Figure 1). In the limit case when the two courts have the same ideal point, all factfinding is helpful. Because factfinding tends to be helpful when ideal points are close together and deceptive when they are far apart, the appellate court prefers the American regime of factfinding discretion only in the former case. Otherwise, the appellate court is better off going by its own signal of case facts, noisy as it is, than having to defer to the trial court's factfinding. This result has an affinity with the "ally principle" that the principal delegates greater discretion to agents with similar preferences (e.g., Gilligan and Krehbiel (1987), Epstein and O'Halloran (1994); but see Bendor and Meirowitz (2004) and Huber and McCarty (2004) for qualifications).

Proposition 4.2 undermines the delegation-in-return-for-effort explanation. If fact deference is beneficial to appellate courts only when trial courts have similar preferences, an explanation grounded in appellate courts' self-interested grant of discretion implies that we should observe more deference when trial and appellate courts tend to agree on the law. But the truth is that factfinding deference is a pervasive feature of the American judicial system, and there are no variations that correspond in any apparent way to preference divergence in the judicial hierarchy. It is unclear why the principal would agree to bind itself categorically to an institution of deference whose benefits are so contingent. The discretion-in-return-for-effort explanation seems unpersuasive because the principal is often better off without the agent's effort.

The failure of this explanation suggests that the search for the institution's *raison d'être* should not stop with intermediate appellate courts. We might do better if, instead of looking to the immediate principal, we look to the principal's principal (e.g., the state or federal Supreme Court) or to a different player in charge of setting the rules of the principal-agent game (e.g., the legislature).

Such an approach would be consistent with evidence of appellate courts' occasional efforts to break free from the shackles of fact deference. A recent high-profile example comes from patents. In patent law, "claim construction" refers to the court's interpretation of a patent "claim," which is the part of a patent application that "defines the scope of the patentee's rights" (*Markman v. Westview Intruments, Inc.*, 517 U.S. 370, 372 (1996)). Claim construction is important in patent litigation, because whether the claim is construed narrowly or broadly often determines whether the defendant has infringed the patentee's rights, and also because an indefinite claim renders the patent invalid (and hence incapable of being infringed) (*Nautilus, Inc. v. Biosig Instruments, Inc.*, 134 S. Ct. 2120 (2014)). It is well-settled that claim construction is, like the construction of contracts and other legal instruments, a question of law (*Markman*, 517 U.S. at 372). But the legal question of claim construction sometimes involves a subsidiary question of determining how a "person of ordinary skill in the art" would understand certain scientific or technical terms, and this latter determination is a factual one (*Teva Pharmaceuticals USA, Inc. v. Sandoz, Inc.*, 135 S. Ct. 831, 838 (2015)). The Federal Circuit, which has exclusive jurisdiction over patent appeals, held in a series of cases that *de novo* review applies not only to the ultimate question of claim construction but also to factual findings made along the way (see *Lighting Ballast Control LLC v. Philips Electronics North Am. Corp.*, 744 F.3d 1272 (Fed. Cir. 2014) (en banc)). The Supreme Court recently reversed this line of cases, holding that a trial judge's resolution of an underlying factual dispute as part of claim construction must be reviewed under the clear-error standard (*Teva Pharmaceuticals*, 135 S. Ct. at 835).

This example demonstrates the importance of standards of review in the context of business and innovation policy. It also shows the need to look beyond intermediate appellate courts in trying to understand why deferential fact review survives. The Federal Circuit sought to increase its hold over patent doctrine by applying a *de novo* standard, producing an "unusually high rate" of reversing district courts (*Teva Pharmaceuticals*, 135 S. Ct. at 839). It was only the Supreme Court's intervention that stemmed this rebellion and ensured the continued vitality of the clear-error standard. Deferential fact review persists not because of the immediate principal's support but in spite of its opposition.

6 Conclusion

I have presented, to my knowledge, the first formal model of the fact-law distinction. The analysis shows that trial courts will not always report case facts truthfully. This result focuses attention on trial courts themselves, not just the parties, as a potential source of fact distortion. As the dropsy example shows, the problem might be of particular concern in criminal justice and policing, areas which have recently attracted much scholarly attention (e.g., Coviello and Persico (2015), Goel, Rao and Shroff (2016), Daughety and Reinganum (2018), Mummolo (2018)). The analysis also shows that appellate courts will not commit to consistent rules. This is a point about the strategy of rulemaking—not its outcome, which always ends up at the appellate court's ideal point in the pure-strategy equilibrium. The distinction between the strategy and outcome of rulemaking lays bare strategic dynamics that would not be empirically observable or conceptually clear without a formal model. The mixed-strategy equilibrium shows how fact discretion can lead to suboptimal (Pareto-dominated) rules, as well as why trial courts may get reversed even when the legal issue is not difficult and appellate review is not discretionary. The most interesting of the comparative statics is that preference divergence has a nonmonotonic effect on factfinding, depending on the kind of factfinding and on the existing level of preference divergence. A comparison of the American judicial system to one without fact deference shows that fact deference is always beneficial to trial courts and beneficial to appellate courts if and only if their preferences are closely aligned with trial courts. This result suggests looking beyond the canonical account of delegation to induce effort in order to understand why the institution of deferential review survives.

These theoretical results can be extended in different directions. In this paper, the trial court's factfinding discretion manifests itself in the choice of facts to report, not in discovering facts; the trial court is assumed to have already learned the facts. As discussed, this modeling choice is appropriate in capturing the American adversarial system. Moreover, learning would occur even absent any strategic principal-agent considerations, and I wanted to focus on strategic factfinding. But the learning and strategic aspects may interact in interesting ways—say, in a model where the trial court also gets a noisy fact signal (but not as noisy as the appellate court's) and can engage in costly factfinding both to reduce its noise and to buy credibility to report facts beyond the appellate court's signal.

Future work can also incorporate legal complexity by assuming that the trial court knows the neighborhood of the appellate court's ideal point but not its exact location. In this modified setting, reversal-averse trial courts would want to use their factfinding discretion to avoid reporting facts in the zone of uncertainty—a form of strategic factfinding distinct from the disposition-motivated variety discussed above, and one which does not require ideal-point divergence. Additional complexities would arise from two-sided uncertainty. Finally, one could consider alternative institutional designs such as factfinding by a panel of trial judges or discretionary factual audits by appellate courts.

The model's comparative statics can be pursued in future empirical work. Two distinct empirical strategies, among others, are worth mentioning. First, in the context of motions to suppress (see Section 4), a trial court may decide the motion on the papers or it may hold a hearing. A suppression hearing imposes additional workload costs, but it widens the range of facts the trial court could permissibly find by giving it an opportunity to hear witnesses and rest its decision on witness credibility. The decision whether to conduct a suppression hearing is thus a good real-world example of costly factfinding as conceptualized in the model, and the comparative statics on factfinding can be tested with the rate of suppression hearings as the dependent variable. Second, one could use text analysis and machine learning to parse out factual from legal discussion in judicial opinions, then use the extent of factual discussion as the dependent variable.

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Appendix

Proof of Remark 1. Suppose for contradiction that $\text{sign}\{f'\} = \text{sign}\{f_t\}\ \forall f$. Then HC would set $r = 0$, so LC would also set $r_\ell = 0$. (That $r = 0$ follows from the fact that 0 is the unique maximizer of the rule component of HC's utility function, and if $\text{sign}\{f'\} = \text{sign}\{f_t\}$ then 0 also maximizes the disposition component. To see that $r_\ell = 0$, note that the last two terms in LC's utility function (equation (1)) are not affected by the choice of r_ℓ, so $U_{LC}(r_\ell = 0) = -L + a - c\varphi + e\mathbb{1}(d = d_L) > -L - c\varphi + e\mathbb{1}(d = d_L) = U_{LC}(r_\ell \neq 0)$.) Consider the region where $f_t \in [0, L)$ and $f \in [0, \epsilon)$. There, $\max\{U_{LC}(f' \geq 0)\} = -L + a < -L + a - c + e = U_{LC}(f' < 0)$. □

Proof of Remark 2. First claim: $f \geq \epsilon \implies \Pr(f_t \geq 0) = 1$, so $r = 0$ uniquely maximizes HC's expected utility. Therefore, $r = r_\ell = 0$ in equilibrium. Fix $r_\ell = 0$ and consider LC's choice of φ. We know that $U_{LC}(\varphi = 0) = -L + a + e\mathbb{1}(d = d_L)$. Given $r = 0$, the last term of LC's payoff is not affected by the choice of φ, so $U_{LC}(\varphi = 1) = -L + a - c + e\mathbb{1}(d = d_L) < U_{LC}(\varphi = 0)$. Second claim: $f \leq -\epsilon \implies \Pr(f_t < 0) = 1$, which implies similarly that $\varphi = 0$. □

Proof of Lemma 1. That $r_\ell = 0$ is a best response was proven for Remark 1. The proof for LC's choices of φ and f' is as follows:

1. If $f_t < L$ then LC can obtain its preferred dispostion without factfinding. If $f_t \geq L$ then $U_{LC}(f' \in [0, f + \epsilon]) = -L + a - c + e > -L + a = \max\{U_{LC}(f' \notin [0, f + \epsilon])\}$. (For the right-hand side of the inequality, note that $U_{LC}(\varphi = 0) = -L + a$ and $U_{LC}(\varphi = 1, f' < 0) = -L + a - c$.)

2. By Remark 2.

3. If $f_t \geq L$ then LC can obtain its preferred dispostion without factfinding. If $f_t < L$ then $U_{LC}(f' \in [f - \epsilon, 0)) = -L + a - c + e > -L + a = \max\{U_{LC}(f' \notin [f - \epsilon, 0))\}$. (For the right-hand side of the inequality, note that $U_{LC}(\varphi = 0) = -L + a$ and $U_{LC}(\varphi = 1, f' \geq 0) = -L + a - c$.) □

Proof of Proposition 1. The strategy of this proof is to check whether $r = 0$ remains HC's best reponse to LC's best-response strategy set forth in

Lemma 1. Suppose $f \in [0, \epsilon)$ and HC observes $f' < 0$. Then by Lemma 1 HC updates the distribution of true facts to $f_t \sim U[f - \epsilon, \min\{L, f + \epsilon\}]$. (This posterior belief assumes that when $f_t < L, LC$'s choice of f' from $\{f' | f' < 0\}$ does not depend on $\text{sign}\{f_t\}$. Finding a profitable deviation from $r = 0$ when f' is independent of f_t is of course sufficient to show that a profitable deviation also exists when f' does depend on f_t.) If HC sticks to the strategy $r = 0$ then $d = 1$ and HC's expected utility is given by $EU_{HC}(r = 0) = e \Pr(d_H = 1) = e \Pr(f_t < 0) = \dfrac{e(\epsilon - f)}{\min\{L, f + \epsilon\} - f + \epsilon}$. Now consider a deviation to $r = f'$. For the deviation to be profitable we must have $EU_{HC}(r = f') > EU_{HC}(r = 0)$, which is to say $f' > \dfrac{e(\epsilon - f - \min\{L, f + \epsilon\})}{\min\{L, f + \epsilon\} - f + \epsilon}$. It is sufficient to check the inequality at $\min f' = f - \epsilon$. And it is easy to see that the inequality is satisfied for values of f sufficiently close to ϵ, because $\lim_{f \uparrow \epsilon} \text{LHS} = 0 > -e = \lim_{f \uparrow \epsilon} \text{RHS}$. We conclude that there exist $f \in [0, \epsilon)$ for which HC has a profitable deviation from $r = 0$, which shows that the strategy $r = 0$ cannot be sustained in equilibrium. $\square$

Proposition 2. *The following profile of strategies and beliefs characterizes a perfect Bayesian equilibrium of the game.*

1. *If $f \geq \epsilon$ or $f \leq -\epsilon$ then HC sets $r = 0$ and LC sets $r_\ell = 0$, $\varphi = 0$.*
   HC's beliefs are that $\Pr(f_t < 0 | f \geq \epsilon) = 0$ and $\Pr(f_t < 0 | f \leq -\epsilon) = 1$.

2. *If $f \in (-\epsilon, 0)$ then*

   - *HC sets $r = 0$.*
   - *LC sets $r_\ell = 0$ and* $\begin{cases} \varphi = 0 & \text{if } f_t < L \\ \varphi = 1, f' \geq 0 & \text{if } f_t \geq L \end{cases}$.
   - *HC's beliefs on path are given by Bayes' rule.*
     Off path, $\Pr(f_t < 0 | \varphi = 1, f' < 0) = 1$ *and* $\Pr(f_t < 0 | f' \geq 0) = 0$.

3. *If $f \in [0, \epsilon)$ then*

   - *HC sets* $r = \begin{cases} 0 & \text{if } EU_{HC}(r = 0) \geq EU_{HC}(r = f') \\ f' & \text{if } EU_{HC}(r = 0) < EU_{HC}(r = f') \end{cases}$.
     In particular, if LC sets $f' \geq 0$ then HC sets $r = 0$ and

$$\textit{if LC sets } f' = f - \epsilon \textit{ then HC sets } r = \begin{cases} 0 & \textit{for } f \leq f^* \\ f - \epsilon & \textit{for } f > f^* \end{cases}$$

$$\textit{where } f^* = \begin{cases} f_m^* & \textit{for } L \geq f_m^* + \epsilon \\ \dfrac{2\epsilon + L + e_h - \sqrt{L^2 + e_h^2 + 6Le_h}}{2} & \textit{for } L < f_m^* + \epsilon \end{cases}$$

$$\textit{and } f_m^* = \frac{\epsilon^2}{e_h + \epsilon}.$$

- $\textit{LC sets } r_\ell = 0 \textit{ and } \begin{cases} \varphi = 1, f' = f - \epsilon & \textit{if } f \leq f^* \textit{ and } f_t < L \\ \varphi = 0 & \textit{otherwise} \end{cases}.$
- *HC's beliefs on path are given by Bayes' rule.*

  Off path, $\Pr(f_t < 0 | f' = x) = \begin{cases} 0 & \textit{if } x \geq 0 \\ \dfrac{\epsilon - f}{\min\{L, f + \epsilon\} - f + \epsilon} & \textit{if } x < 0 \end{cases}$

  (that is, if $f' < 0$ then HC believes that $f_t \sim U[f - \epsilon, \min\{L, f + \epsilon\}]$).

Proof. For $f \geq \epsilon$ or $f \leq -\epsilon$, that the strategy-belief profile is an equilibrium follows immediately from Remark 2. For other values of f, it is convenient to consider three cases separately: (1) $L \geq 2\epsilon$ (the two courts' ideal points are very far apart), (2) $L \in [\epsilon, 2\epsilon)$ (ideal points are moderately far apart), (3) $L < \epsilon$ (ideal points are close).

Case 1: $L \geq 2\epsilon$. In this case, parts 2-3 of the Proposition become:

2. If $f \in (-\epsilon, 0)$ then HC sets $r = 0$ and LC sets $r_\ell = 0, \varphi = 0$.

3. If $f \in [0, \epsilon)$ then HC's strategy is as described in the Proposition and

   LC sets $r_\ell = 0$ and $\begin{cases} \varphi = 1, f' = f - \epsilon & \text{if } f \leq f_m^* \\ \varphi = 0 & \text{otherwise} \end{cases}.$

First consider $f \in (-\epsilon, 0)$. That LC's strategy is a best response was proven in Lemma 1. As for HC: On the equilibrium path, note that $r > f$ is strictly dominated by $r = 0$ and $r < f$ is strictly dominated by $r = f$. Because LC is always setting $\varphi = 0$, HC's posterior is the same as its prior: $f_t \sim [f - \epsilon, f + \epsilon]$. So $EU_{HC}(r = 0) = \dfrac{e(\epsilon - f)}{2\epsilon} > f + \dfrac{e(\epsilon + f)}{2\epsilon} = EU_{HC}(r = f)$, so $r = 0$ is a best response. Off the equilibrium path, if $\varphi = 1$ and $f' < 0$ then, given off-path beliefs, $EU_{HC}(r = 0) = e = \max U_{HC}$. And if $f' \geq 0$ then, given off-path beliefs, $EU_{HC}(r = 0) = e = \max U_{HC}$.

Next consider $f \in [0, \epsilon)$. First we verify that HC's strategy is a best response if LC sets $f' < 0$. Note that $r > f'$ is strictly dominated by $r = 0$, and $r < f'$ is strictly dominated by $r = f'$, so the best rule choice is either f' or 0. Now consider in particular the case $f' = f - \epsilon$. Let $g(f) \equiv EU_{HC}(r = 0) - EU_{HC}(r = f - \epsilon) = \dfrac{\epsilon^2 - f(e + \epsilon)}{\epsilon}$ and solve for f when $g(f) = 0$, which yields $f = \dfrac{\epsilon^2}{e + \epsilon} \equiv f_m^*$. Note that $\partial g / \partial f < 0$, so HC's best response is to choose $r = 0$ when $f < f_m^*$ and $r = f - \epsilon$ when $f > f_m^*$.

Next we verify that HC's strategy is a best response if LC does not engage in factfinding. If $\varphi = 0$ and $f > f_m^*$ then by Bayes' rule there is no updating of $f_t \sim U[f - \epsilon, f + \epsilon]$, so $\Pr(f_t \geq 0) = \dfrac{\epsilon + f}{2\epsilon} \geq \dfrac{\epsilon - f}{2\epsilon} = \Pr(f_t < 0)$. Therefore, $EU_{HC}(r = 0) = e \Pr(f_t \geq 0) > -r + e \Pr(f_t < 0) = EU_{HC}(r > f)$. We also have that $e \Pr(f_t \geq 0) \geq -|r| + e \Pr(f_t \geq 0) = EU_{HC}(r \leq f)$. If $\varphi = 0$ and $f \leq f_m^*$ then (given off-path beliefs) we have $EU_{HC}(r = 0) = e = \max U_{HC}$. Finally we verify that HC's strategy is a best response if $\varphi = 1$ and $f' \geq 0$. In that case, off-path beliefs dictate that $\Pr(f_t < 0) = 1$, so $EU_{HC}(r = 0) = e = \max U_{HC}$.

As for LC, note first that $L \geq 2\epsilon$ implies $f_t < L$. If $f \leq f_m^*$ then LC's strategy of $f' = f - \epsilon$ yields $U_{LC} = -L + e - c + a$. As for deviations, $U_{LC}(r_\ell = 0, \varphi = 0) = -L + a$; $U_{LC}(r_\ell \neq 0, \varphi = 0) = -L$; $U_{LC}(r_\ell = 0, \varphi = 1, f' \geq 0) = -L - c + a$; $U_{LC}(r_\ell \neq 0, \varphi = 1, f' \geq 0) = -L - c$; $\max U_{LC}(r_\ell = 0, f' < 0) = -L + e - c + a$; $U_{LC}(r_\ell = f', f' < 0) = \begin{cases} -L + e - c & \text{if } r = 0 \\ f' - L - c + a & \text{if } r = f' \end{cases}$; and $\max U_{LC}(r_\ell \notin \{0, f'\}, f' < 0) = -L + e - c$. We see that there are no profitable deviations and LC's strategy is a best response when $f \leq f_m^*$.

If $f > f_m^*$ then LC's strategy of $r_\ell = 0, \varphi = 0$ yields $U_{LC} = -L + a$. As for deviations, $\max U_{LC}(f' < 0) = f' - L - c + a$; $\max U_{LC}(\varphi = 1, f' \geq 0) = -L - c + a$; and $U_{LC}(r_\ell \neq 0, \varphi = 0) = -L$. So there are no profitable deviations and LC's strategy is a best response when $f > f_m^*$. This completes the proof for Case 1.

Case 2: $L \in [\epsilon, 2\epsilon)$. The proof for when $f \in (-\epsilon, 0)$ is the same as in the last case, so focus on $f \in [0, \epsilon)$. First we show that HC's strategy is a best response when $f' < 0$. It is clear, as before, that all rule choices except for $r = 0$ and $r = f'$ are strictly dominated, so the general form of HC's strategy is correct. More particularly, we must show that HC's strategy is a best response when $f' = f - \epsilon$. The strategy of the proof is, first, to establish that the threshold value f_m^* calculated above remains the minimal threshold below which factfinding will be tolerated, and then to investigate whether and under what conditions factfinding would also be tolerated for f above f_m^*. First we show that if $f \leq f_m^*$ and $f' = f - \epsilon$ then $r = 0$ is a best response. We know that $r = 0$ is a best response whenever $EU_{HC}(r = 0) \geq EU_{HC}(r = f - \epsilon)$, which is to say $e \Pr(f_t < 0) \geq f - \epsilon + e \Pr(f_t \geq 0)$. Given HC's beliefs when $f' < 0$, this condition can be written as

$$\frac{-ef}{\epsilon} \geq f - \epsilon \qquad \text{when } L \geq 2\epsilon \tag{4}$$

$$\frac{e(\epsilon - f)}{\min\{L, f + \epsilon\} - f + \epsilon} \geq f - \epsilon + \frac{e \min\{L, f + \epsilon\}}{\min\{L, f + \epsilon\} - f + \epsilon} \qquad \text{when } L < 2\epsilon \tag{5}$$

We know from Case 1 that condition (4) is satisfied for all $f \leq f_m^*$, so to show that condition (5) is also satisfied, it is sufficient to show that (4) $\implies$ (5). If $L \geq f + \epsilon$ then conditions (4) and (5) are the same and we are done. If $L < f + \epsilon$ then condition (5) can be rewritten as

$$\frac{e(\epsilon - f - L)}{L - f + \epsilon} \geq f - \epsilon \tag{6}$$

To show that (4) $\implies$ (6) it is sufficient to show that $\dfrac{-ef}{\epsilon} \leq \dfrac{e(\epsilon - f - L)}{L - f + \epsilon}$, which after algebra reduces to $\epsilon + f - L \geq 0$, which is satisfied by hypothesis. (Moreover, note that when $L < f + \epsilon$, if (4) holds weakly then (6) holds strictly.) Recalling from Case 1 that Condition (4) is equivalent to $f \leq f_m^*$, we conclude that whenever $f \leq f_m^*$, a best response of HC to $f' = f - \epsilon$ is $r = 0$.

Next we consider whether HC would also set $r = 0$ in response to $f' = f - \epsilon$ when $f > f_m^*$. First consider the case $L < f_m^* + \epsilon$. Then $f \in (f_m^*, \epsilon) \implies f + \epsilon > L$, so $EU_{HC}(r = 0) \geq EU_{HC}(r = f - \epsilon)$ iff

$$f^2 - f(2\epsilon + L + e) + \epsilon^2 + L\epsilon + e\epsilon - eL \geq 0 \tag{7}$$

Denoting the LHS of (7) by $g(f)$, it is straightforward to verify that $\partial g / \partial f < 0$, $g(\epsilon) < 0$, and $g(f_m^*) > 0$. It follows from the intermediate value theorem that

$\exists! f^* \in (f_m^*, \epsilon)$ s.t. $g(f^*) = 0$. Solving for this unique f^*, we obtain

$$f^* = \frac{2\epsilon + L + e - \sqrt{L^2 + e^2 + 6eL}}{2}$$

(It is easy to verify algebraically that $f^* \in (f_m^*, \epsilon)$ $\forall L \in [\epsilon, f_m^* + \epsilon)$, and that $f^*(L)\Big|_{L=f_m^*+\epsilon} = f_m^*$.) We conclude that whenever $L < f_m^* + \epsilon$ and $f \leq f^*$, a best response of HC to $f' = f - \epsilon$ is $r = 0$.

Next consider the case $L \geq f_m^* + \epsilon$. There are two cases to consider:
(a) $f \leq L - \epsilon$, (b) $f > L - \epsilon$. In case (a), if LC sets $f' = f - \epsilon$ then HC updates to $f_t \sim U[f - \epsilon, f + \epsilon]$ and, because $f > f_m^*$, we know from Case 1 (p. 43) that $EU_{HC}(r = 0) < EU_{HC}(r = f - \epsilon)$. Therefore, HC would set $r = f'$ if LC sets $f' = f - \epsilon$ (or any $f' < 0$). In case (b), if LC sets $f' = f - \epsilon$ then HC updates to $f_t \sim U[f - \epsilon, L)$ and we have that $EU_{HC}(r = 0) \geq EU_{HC}(r = f - \epsilon)$ iff

$$\frac{e(\epsilon - f)}{L - f + \epsilon} \geq f - \epsilon + \frac{eL}{L - f + \epsilon} \tag{8}$$

Now recall that at f_m^* we have

$$\frac{e(\epsilon - f_m^*)}{2\epsilon} = f_m^* - \epsilon + \frac{eL}{2\epsilon} \tag{9}$$

Note that $LHS(8) < LHS(9)$ and $RHS(8) > RHS(9)$, so (8) cannot hold. Therefore, when $L \geq f_m^* + \epsilon$ and $f > f_m^*$, HC's best response to $f' = f - \epsilon$ is $r = f - \epsilon$.

The proof so far has derived the factfinding threshold f^* specified in the Proposition, which is a function of L for $L < f_m^* + \epsilon$ and stabilizes at f_m^* for $L \geq f_m^* + \epsilon$. We have shown that HC would not tolerate factfinding when $f > f^*$ but would tolerate factfinding at $f' = f - \epsilon$ when $f \leq f^*$. We conclude that HC's strategy is a best response when $f' < 0$.

Next we show that $r = 0$ is HC's best response when $f' \geq 0$. If $\varphi = 1$ and $f' \geq 0$ then, given off-path beliefs, $\Pr(f_t < 0) = 0$ and $EU_{HC}(r = 0) = e = \max U_{HC}$. If $\varphi = 0$ and $f > f^*$ then $r \leq f$ is dominated by $r = 0$ and, for $r > f$, EU_{HC} is decreasing in r. By Bayes' rule HC's posterior is the same as its prior, so $EU_{HC}(r = 0) = e(\epsilon + f)/2\epsilon > -f + e(\epsilon - f)/2\epsilon = \lim_{x \downarrow f} EU_{HC}(r = x)$, so $r = 0$ is a best response. If $\varphi = 0$ and $f < f^*$ then $\Pr(f_t < 0) = 0$, so

$EU_{HC}(r=0) = e = \max U_{HC}$. It follows that $r = 0$ is a best response whenever $f' \geq 0$. We conclude that HC's strategy is a best response.

Next we show that LC's strategy is a best response. If $f_t < L$ then LC's strategy is a best response by the same reasoning as in Case 1 (p. 43). If $f_t \geq L$ and $f \leq f^*$ then LC's strategy of $r_\ell = 0, \varphi = 0$ yields $U_{LC} = -L + e + a$. Now consider deviations:
$\max U_{LC}(\varphi = 1, f' \geq 0) = -L + e - c + a$; $U_{LC}(r_\ell \neq 0, \varphi = 0) = -L + e$; $U_{LC}(r_\ell = 0, f' < 0) = \begin{cases} -L - c + a & \text{if } r = 0 \\ f' - L - c + e & \text{if } r = f' \end{cases}$; $U_{LC}(r_\ell = f', f' < 0) = \begin{cases} -L - c & \text{if } r = 0 \\ f' - L + e - c + a & \text{if } r = f' \end{cases}$; and $U_{LC}(r_\ell \notin \{0, f'\}, f' < 0) = \begin{cases} -L - c & \text{if } r = 0 \\ f' - L + e - c & \text{if } r = f' \end{cases}$.
None of these deviations are profitable, so LC's strategy is a best response.

Finally consider $f_t \geq L$ and $f > f^*$. LC's strategy of $r_\ell = 0, \varphi = 0$ yields $U_{LC} = -L + e + a$. As for deviations, $\max U_{LC}(\varphi = 1, f' \geq 0) = -L + e - c + a$; $U_{LC}(r_\ell \neq 0, \varphi = 0) = -L + e$; and $\max U_{LC}(f' < 0) = f' - L + e - c + a$. None of these deviations are profitable, so LC's strategy is a best response. This concludes the proof for Case 2.

Case 3: $L < \epsilon$. The proof for when $f \in [0, \epsilon)$ is the same as in Case 2. Focus on $f \in (-\epsilon, 0)$. That LC's strategy is a best response was shown in the proof of Lemma 1. As for HC, if $f' \geq 0$ then $\Pr(f_t < 0) = 0$ and $EU_{HC}(r = 0) = e = \max U_{HC}$. If $\varphi = 1$ and $f' < 0$ then, given off-path beliefs, $\Pr(f_t < 0) = 1$ and $EU_{HC}(r = 0) = e = \max U_{HC}$. Finally, if $\varphi = 0$ then all rule choices except for $r = 0$ and $r = f$ are dominated. Consider two cases separately: (a) $f < L - \epsilon$, (b) $f \geq L - \epsilon$. In case (a), $EU_{HC}(r = 0) = \dfrac{e(\epsilon - f)}{2\epsilon} > f + \dfrac{e(\epsilon + f)}{2\epsilon} = EU_{HC}(r = f)$. And in case (b), $EU_{HC}(r = 0) = \dfrac{e(\epsilon - f)}{L - f + \epsilon} > f + \dfrac{eL}{L - f + \epsilon} = EU_{HC}(r = f)$. It follows that HC's strategy is a best response. We have shown that the strategy-belief profile in Proposition 2 characterizes a perfect Bayesian equilibrium. □

Proposition 3. *The following profile of strategies and beliefs characterizes a perfect Bayesian equilibrium of the game when $L \geq 2\epsilon$.*

1. *If $f \geq \epsilon$ or $f \leq -\epsilon$ then HC sets $r = 0$ and LC sets $r_\ell = 0$, $\varphi = 0$.*
   HC's beliefs are that $\Pr(f_t < 0 | f \geq \epsilon) = 0$ and $\Pr(f_t < 0 | f \leq -\epsilon) = 1$.

2. *If $f \in (-\epsilon, 0)$ then HC sets $r = 0$ and LC sets $r_\ell = 0$, $\varphi = 0$.*

   HC's beliefs on path are given by Bayes' rule.

   Off path, $\Pr(f_t < 0 | \varphi = 1, f' < 0) = 1$ *and* $\Pr(f_t < 0 | f' \geq 0) = 0$.

3. *If $f \in [0, f_m^*]$ then*

   - *HC sets* $r = \begin{cases} 0 & \text{if } EU_{HC}(r=0) \geq EU_{HC}(r=f') \\ f' & \text{if } EU_{HC}(r=0) < EU_{HC}(r=f') \end{cases}$.

     In particular, if LC sets $f' = f - \epsilon$ then HC sets $r = 0$.

   - *LC sets $r_\ell = 0$, $\varphi = 1$, $f' = f - \epsilon$.*

   - *HC's beliefs on path by Bayes. Off path,* $\Pr(f_t < 0 | f' = x) = \begin{cases} 0 & \text{if } x \geq 0 \\ \dfrac{\epsilon - f}{2\epsilon} & \text{if } x < 0 \end{cases}$.

4. *If $f \in (f_m^*, \epsilon)$ then*

   - *HC's strategy is:*

     – *If $r_\ell \neq 0$ then* $r = \begin{cases} 0 & \text{if } f' \geq 0 \\ f' & \text{if } f' < 0 \end{cases}$.

     – *If $r_\ell = 0$ then* $\begin{cases} r = 0 & \text{if } f' \geq 0 \\ r = f' & \text{if } f' \in (f - \epsilon, 0) \\ r = \begin{cases} 0 & \text{w/ prob. } p \\ f' & \text{w/ prob. } 1 - p \end{cases} & \text{if } f' = f - \epsilon \end{cases}$

       where $p = \dfrac{a + c + \epsilon - f}{a + e_\ell + \epsilon - f}$.

   - *LC sets $r_\ell = 0$ and*

     – *if $f_t < 0$ then* $\begin{cases} \varphi = 0 & \text{w/ prob. } 1 - \pi_1 \\ \varphi = 1, f' = f - \epsilon & \text{w/ prob. } \pi_1 \end{cases}$

     – *if $f_t \geq 0$ then* $\begin{cases} \varphi = 0 & \text{w/ prob. } 1 - \pi_2 \\ \varphi = 1, f' = f - \epsilon & \text{w/ prob. } \pi_2 \end{cases}$

       where $\pi_2 = \left(\dfrac{\epsilon - f}{\epsilon + f}\right)\left(\dfrac{\epsilon - f + e_h}{f - \epsilon + e_h}\right)\pi_1$.

   - *HC's beliefs on the equilibrium path are given by Bayes' rule. Off path,* $\Pr(f_t < 0 | r_\ell \neq 0, f' < 0) = 0$, $\Pr(f_t < 0 | \varphi = 1, f' \geq 0) = 0$,

$\Pr(f_t < 0 | r_\ell = 0, f' \in (f-\epsilon, 0)) = \Pr(f_t < 0 | r_\ell = 0, f' = f - \epsilon)$, *and* $\Pr(f_t < 0 | r_\ell \neq 0, \varphi = 0) = \Pr(f_t < 0 | r_\ell = 0, \varphi = 0)$. *(The last two equations indicate that off-path beliefs are the same as on-path beliefs in analogous situations.)*

Proof. The only new result that requires proof is that the parties' strategies are best responses when $f \in (f_m^*, \epsilon)$ (for other values of f, see Case 1 in the proof of Proposition 2). First consider LC's strategy. For LC to randomize between not engaging in factfinding and factfinding at $f' = f - \epsilon$, HC must set the probability of reversal (given by $1 - p$) such that LC would be indifferent between the two choices. Setting $U_{LC}(r_\ell = 0, \varphi = 0) = EU_{LC}(r_\ell = 0, f' = f - \epsilon)$ and solving for p yields $p = \dfrac{a + c + \epsilon - f}{a + e_\ell + \epsilon - f}$. LC's expected utility from this strategy is $-L + a$. Now consider deviations:

$U_{LC}(r_\ell = 0, \varphi = 1, f' \geq 0) = -L - c + a$; $U_{LC}(r_\ell = 0, f' \in (f - \epsilon, 0)) = f' - L - c$;
$U_{LC}(r_\ell \neq 0, \varphi = 0) = -L$; $U_{LC}(r_\ell \neq 0, \varphi = 1, f' \geq 0) = -L - c$;
$U_{LC}(r_\ell = f', f' < 0) = f' - L - c + a$; and $U_{LC}(r_\ell \notin \{0, f'\}, f' < 0) = f' - L - c$.
There are no profitable deviations, so LC's strategy is a best response.

Next consider HC's strategy. First consider $r_\ell = 0, f' = f - \epsilon$. As before, all rule choices except for $r = 0$ and $r = f - \epsilon$ are dominated. To randomize between $r = 0$ and $r = f - \epsilon$, HC must be indifferent between the two. Setting $EU_{HC}(r = 0) = EU_{HC}(r = f - \epsilon)$ and solving for LC's probability of factfinding, we obtain $\pi_2 = \pi_1 \left(\dfrac{\epsilon - f}{\epsilon + f}\right)\left(\dfrac{\epsilon - f + e_h}{f - \epsilon + e_h}\right)$ where $\pi_1 = \Pr(f' = f - \epsilon | f_t < 0)$ and $\pi_2 = \Pr(f' = f - \epsilon | f_t \geq 0)$. (It is straightforward to verify that π_2 is a proper probability if π_1 is a proper probability, and that $\pi_2 < \pi_1$.) We conclude that HC's strategy is a best response when $r_\ell = 0$ and $f' = f - \epsilon$.

Next consider $r_\ell = 0, f' \in (f - \epsilon, 0)$. Given off-path beliefs, HC's posterior about the distribution of f_t is the same as in the last case where $f' = f - \epsilon$, so $EU_{HC}(r = f') > EU_{HC}(r = f - \epsilon) = EU_{HC}(r = 0)$ (where the inequality follows from the fact that $f' > f - \epsilon$). Noting that all choices other than 0 and f' are dominated, it follows that $r = f'$ is a best response.

Next consider $r_\ell \neq 0, f' < 0$. The necessary and sufficient condition for $r = f'$ to be a best response is $EU_{HC}(r = f') \geq EU_{HC}(r = 0)$, which is to say $f' + e \geq 0$. It is sufficient to check the inequality at $\inf f' = f_m^* - \epsilon$, where it is satisfied.

Next consider $r_\ell = 0, \varphi = 0$. The necessary and sufficient condition for $r = 0$ to be a best response is $EU_{HC}(r = 0) \geq \lim_{x \downarrow f} EU_{HC}(r = x)$, which is to say

$$\frac{e(1-\pi_2)(\epsilon+f)}{(1-\pi_1)(\epsilon-f)+(1-\pi_2)(\epsilon+f)} \geq \frac{e(1-\pi_1)(\epsilon-f)}{(1-\pi_1)(\epsilon-f)+(1-\pi_2)(\epsilon+f)} - f.$$

Because $\pi_1 > \pi_2$ and $\epsilon + f > \epsilon - f$, the inequality is always satisfied.

Next consider $r_\ell \neq 0, \varphi = 0$. Posterior probabilities are the same as in the previous case, so by the same logic we conclude that $r = 0$ is a best response. Finally consider $\varphi = 1, f' \geq 0$. Given off-path beliefs, $EU_{HC}(r = 0) = e = \max U_{HC}$. We conclude that HC's strategy is a best response. $\square$

Derivation of Comparative Statics. To ascertain the impact of parameter changes on factfinding or factfudging, one must first find expressions for the probability of factfinding or factfudging in a given equilibrium and then differentiate those expressions with respect to the parameter of interest. For example, suppose we want to know how factfinding, factfudging, and helpful factfinding change with ϵ when $L \in [\epsilon, f_m^* + \epsilon)$ (Case 2 in Proposition 2). First we verify that when $L \in [\epsilon, f_m^* + \epsilon)$, factfinding occurs iff $f \in (0, f*]$ and $f_t < L$. Then we compute the impact of a change in ϵ on the probability of factfinding, which is given by

$$\frac{\partial}{\partial \epsilon}\left\{\int_0^{L-\epsilon} f_f(x)dx + \int_{L-\epsilon}^{f^*}\int_{x-\epsilon}^{L} f_{f_t|f}(y|x)f_f(x)dydx\right\} = \frac{\partial}{\partial \epsilon}\left\{\int_0^{L-\epsilon} f_f(x)dx + \int_{L-\epsilon}^{f^*}\int_{x-\epsilon}^{L} \frac{1}{2\epsilon}f_f(x)dydx\right\}.$$

Similarly, the impact of ϵ on factfudging when $L \in [\epsilon, f_m^* + \epsilon)$ is given by

$$\frac{\partial}{\partial \epsilon}\left\{\int_0^{L-\epsilon}\int_0^{x+\epsilon} \frac{1}{2\epsilon}f_f(x)dydx + \int_{L-\epsilon}^{f^*}\int_0^{L} \frac{1}{2\epsilon}f_f(x)dydx\right\}$$

and the impact of ϵ on helpful factfinding is given by

$$\frac{\partial}{\partial \epsilon}\int_0^{f^*}\int_{x-\epsilon}^{0} \frac{1}{2\epsilon}f_f(x)dydx.$$

Similar expressions for probabilities of factfinding, factfudging, and helpful factfinding were found for all regions of L (Cases 1-3 in the proof of Proposition 2), and the effects of ϵ, e_h, and L were obtained by differentiation. Figuring out the sign of the derivative was tedious or tricky in some cases, but the calculations involve no more than the application of calculus and algebra.

Proof of Proposition 4. To determine whether LC and HC are better off with factfinding discretion, I compare their equilibrium utilities in the present game to equilibrium utilities in a modified setup where LC is obligated to report $f' = f$. In the latter setup, the factfinding component of LC's strategy is obligatory; HC has the strictly dominant strategy $r = 0$ (given the distribution of $f_t|f$); and, given HC's strategy, LC sets $r_\ell = 0$ in equilibrium. So the equilibrium strategy profile is the same regardless of L, and equilibrium utilities can easily be calculated. These are then compared to equilibrium utilities in Proposition 2. For these comparisons it is assumed naturally that f is distributed uniformly on a large interval around H, which I denote by $f \sim U[-z, z]$ for some $z > 2\epsilon$.[21]

It is clear that LC is always better off with factfinding discretion than without. HC is better off with factfinding discretion than without iff

$$\Pr(f \in (L-\epsilon, 0) \cap f_t \geq L)) + \Pr(f \in (0, f^*) \cap f_t < 0) > \Pr(f \in (0, f^*) \cap f_t \in [0, L))$$

which is to say

$$\int_{L-\epsilon}^{0} \int_{L}^{x+\epsilon} \frac{1}{4\epsilon z} dy dx + \int_{0}^{f^*} \int_{x-\epsilon}^{0} \frac{1}{4\epsilon z} dy dx > \int_{0}^{f^*} \int_{0}^{L} \frac{1}{4\epsilon z} dy dx$$

which after algebra reduces to $\dfrac{(L-\epsilon)^2}{2} + \epsilon f^* - \dfrac{f^{*2}}{2} - Lf^* > 0$. Denoting the LHS by $g(L)$, note that $g(0) = \epsilon^2 > 0$ and $g(\epsilon) = -f^{*2}/2 < 0$, and it can be verified that $\partial g / \partial L < 0$. It follows from the intermediate value theorem that $\exists! \tilde{L} \in (0, \epsilon)$ such that $g(L) > 0$ for $L < \tilde{L}$ and $g(L) < 0$ for $L > \tilde{L}$. We conclude that HC is better off with factfinding than without iff $L < \tilde{L}$. $\square$

[21] Any other distributional assumption would "rig" the comparison by making it an artifact of the fact that f is more likely to fall in certain regions, rather than being strictly a comparison between different judicial factfinding regimes. But, if desired, the comparison could be carried out just as easily for any distribution of f. The result that LC is better off with factfinding discretion than without goes through for any distribution of f.