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PRICE CAPS IN MULTI-PRICE MARKETS

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Abstract

Many consumer markets feature a multi-dimensional price. A policymaker – a legislator, a regulator or a court – concerned about the level of one price dimension may decide to cap this price. How will such a price cap affect other price dimensions? Will the overall effect be good or bad for consumers? For social welfare? Price caps can be beneficial when sellers set prices in response to consumer misperception. The scope for welfare-enhancing regulation depends on the type (and direction) of the underlying misperception, as well as on market structure.

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1. Introduction

A. Motivation

The Credit Card Accountability, Responsibility and Disclosure Act of 2009 (the CARD Act), and its implementing regulations, imposed restrictions on certain dimensions of the credit card price. In particular, late fees were subjected to a de facto price cap. Other fees and interest rates were also curtailed. A few years later, the Dodd-Frank Act restricted the permissible magnitude of prepayment penalties in mortgage contracts. Other examples of price caps are easy to find. Usury laws cap interest rates. Courts applying the Penalty Doctrine imposed de facto caps on cellphone early termination fees. The European Union caps roaming fees and international calling rates. The Singapore Telecommunication Act of 2000 caps the price that hotels can charge for international phone calls. Etc.

In these examples, lawmakers, responding to concern about an excessively high price, resolved to cap the suspect price. The lawmakers did not fully account, however, for the possibility of unintended consequences. In particular, credit cards, mortgages, cellular service and hospitality services are all multi-dimensional products with multi-dimensional prices. When the law caps one price dimension, we cannot assume that other price dimensions will remain unchanged. If sellers react to the new law by increasing other prices, then it is no longer clear that the law will achieve its stated purpose.

Will the price cap increase social welfare? Will it make consumers better off? To answer these questions we need to first understand the forces driving the pre-cap pricing structure. If prices were efficient, designed to provide optimal incentives, then a price cap will likely reduce social welfare and hurt consumers. These distortions might be

exacerbated in a multi-price market, where a price cap on one dimension can lead to adjustment away from the efficient level also on other price dimensions. If, on the other hand, pre-cap prices were designed not to maximize efficiency but to exploit consumer biases, then legal intervention may increase welfare and help consumers.

B. Framework

I study a model with two product, and price, dimensions: The first dimension, labeled Dimension X, represents a binary decision – to purchase, or not to purchase the product. Think of a decision whether or not to book a room for one week in a certain hotel (the duration of your stay is determined exogenously). The consumer enjoys a base utility from the hotel room and pays a base price, p_x, for the hotel room. The second dimension, Dimension Y, represents a continuous use decision. Think of a decision to order room service – the consumer could order room service for any number of meals during her stay at the hotel. The consumer enjoys a per-use, or per-meal, utility from in-room dining and pays a per-use, or per-meal, price, p_y.

Misperception afflicts only the use dimension (Dimension Y). I consider two types of consumer misperception: utility misperception and price misperception. With utility misperception, the consumer under- or overestimates the utility from Dimension Y. For example, the consumer might underestimate the utility from in-room dining. With price misperception, the consumer under- or overestimates the per-use price, p_y. For example, when booking the hotel room, the consumer might underestimate the room-service prices in the hotel.

C. Second-Best Optimal Prices

When demand is biased by consumer misperception, pre-cap prices will be inefficient. To assess this inefficiency, it is helpful to first characterize the efficient prices in a multi-dimensional pricing scheme. I refer to these prices as second-best optimal, i.e., optimal given the misperception. (This paper considers different types, directions and levels of misperception, but the misperception is exogenous to the analysis. For a discussion of endogenous misperceptions – see, e.g., Bar-Gill, 2012.) In the absence of misperception, cost-based pricing is efficient: each price dimension should be set equal to the cost of providing the corresponding product or service dimension. When misperception is introduced, cost-based pricing is no longer optimal.

Interestingly, while the second-best optimal prices depend on the type of misperception – utility misperception vs. price misperception – they do not depend on the direction of the misperception. Optimal prices are the same for both under- and overestimation. And this result holds for both utility misperception and price misperception.

Utility misperception. With utility misperception, it is second-best optimal to increase the price associated with the misperceived dimension, p_y, above cost and reduce the price associated with the accurately perceived dimension, p_x, below cost. This pricing pattern holds for both under- and overestimation of utility, but for different reasons. When consumers underestimate the utility from in-room dining, demand for hospitality services will be too low (since in-room dining is one dimension in the bundle of services provided by the hotel). Demand can be efficiently increased by shifting pricing towards the underestimated dimension, i.e., by increasing room-service prices and reducing the

base-rate charged for the hotel room itself. A consumer who underestimates the utility from in room-dining will also underestimate the number of in-room meals that she will order. Accordingly, the consumer will underestimate the effect of higher room-service prices. While the reduction in p_x will be fully appreciated, the effect of the corresponding increase in p_y will be underestimated. Therefore, the perceived total price will go down, and demand will go up.

When consumers overestimate the utility from in-room dining, demand for hospitality services will be too high. But, again, increasing room-service prices and reducing the base-rate improve things – this time by reducing demand. A consumer who overestimates the utility from in room-dining will also overestimate the number of in-room meals that she will order. Accordingly, the consumer will overestimate the effect of higher room-service prices. The consumer will accurately perceive the reduction in p_x, and overestimate the effect of the corresponding increase in p_y. Therefore, the perceived total price will go up, and demand will go down.

Price misperception. With price misperception, second-best prices will, again, be independent of the direction of the misperception. But these second-best prices will be very different from those obtained for utility misperception: It is optimal to reduce the misperceived price, p_y, below cost and to increase the accurately perceived price, p_x, above cost. With both under- and overestimation, it is optimal to shift pricing away from the misperceived price dimension and towards the accurately perceived price dimension. Such a shift reduces the difference between the actual total price that the consumer will pay for the product and the price that the consumer thinks she will pay for the product.

D. Pre-cap Equilibrium Prices

To assess the welfare cost of misperception, and the scope for welfare-enhancing price caps, I now compare the second-best prices to the pre-cap, equilibrium prices. I begin with perfect competition and then consider the implications of monopoly and market power.

We saw that the second-best prices depend on the type of misperception, but not on the direction of the misperception. In contrast, the pre-cap, equilibrium prices are very much affected by the direction of the misperception, but not so much by the type of misperception. For both utility underestimation and price underestimation, the equilibrium pre-cap, per-use price, p_y, exceeds the per-use cost; and the equilibrium base-price, p_x, is below the base cost. In the hotel example, in-room dining prices are set above the cost to the hotel of providing this service, while the basic room rates are set below cost. Conversely, for both utility overestimation and price overestimation, the pre-cap, per-use price, p_y, is below the per-use cost; and the base-price, p_x, exceeds the base cost.

Underestimation. When the utility from in-room dining is underestimated, we saw that the second-best room-service prices are above cost. The pre-cap equilibrium room-service prices will be even higher. By reducing the basic room rates and increasing room-service prices, the hotel increases the perceived (net) value of its product and counteracts the utility underestimation. But, at the same time, the hotel reduces the actual value by distorting incentives to utilize its in-room dining services – high room-service prices imply fewer room-service orders. Second-best optimal pricing balances these two effects.

The hotel, however, cares only about perceived value and thus sets room-service prices inefficiently high.

When room-service prices themselves are underestimated, the pre-cap equilibrium room-service prices will again be above cost. As with utility underestimation, the hotel increases the perceived (net) value of its product by shifting prices towards the underestimated dimension. Note, however, that the difference between the equilibrium price and the second-best price, and correspondingly the space for a welfare-enhancing price cap, is significantly larger with price underestimation. While the equilibrium price is above-cost for both types of misperception, the second-best price is significantly lower with price misperception.

Overestimation. When the utility from in-room dining is overestimated, the consumer overestimates the number of room-service orders and thus overestimates the importance of room-service prices. To minimize the perceived total price of its product, the hotel responds to the misperception by reducing room-service prices below cost, and increasing the basic room rate above cost. Similarly, when room-service prices themselves are overestimated, the hotel shifts prices away from the overestimated, room-service dimension and to the accurately perceived basic room rate.

When utility from in-room dining or the price of in-room dining are overestimated, room-service prices are too low and there is no point in capping them. But, since pricing will shift to the accurately perceived basic room rate, this price will be too high and welfare can potentially be enhanced by capping it. While the pre-cap equilibrium room rate is similarly above cost for both utility and price overestimation, second-best prices are very different. Specifically, the second-best room rate is significantly lower with

utility overestimation and, accordingly, there is more room for a welfare-enhancing price cap when the object of misperception is utility rather than price.

Summary. The nature and scope of welfare-enhancing regulation depends on both the type and direction of misperception. With underestimation – of both utility and price – it is the misperceived price (or the price associated with the misperceived dimension), p_y, that needs to be capped. With overestimation it is the accurately perceived price (or the price associated with the accurately perceived dimension), p_x, that needs to be capped.

The question of scope is of particular importance when imperfectly informed lawmakers might set the price cap too low and reduce welfare. When the problem is underestimation, price misperception gives the imperfectly informed lawmaker a larger target to aim at. In particular, with price underestimation, the lawmaker can set the cap at cost (since the pre-cap equilibrium price is above cost and the second-best price is below cost), when information about cost is more readily available. With utility underestimation, a price cap equal to cost is too low (since both the pre-cap equilibrium price and the second-best price are above cost). Conversely, when the problem is overestimation, utility misperception gives the lawmaker a bigger target. In particular, with utility overestimation the lawmaker can set the cap at cost, whereas such a cap might reduce welfare with price overestimation.

E. Market Power

I have thus far assumed that sellers operate in a perfectly competitive market. I now replace competition with monopoly and examine how market power alters the positive

and normative implications of price regulation. The model developed in this paper assumes that the X and Y dimensions are separable. Therefore, the price associated with the misperceived dimension, p_y, does not depend on market structure, and many of the results derived in the perfect competition case apply in the monopoly case as well. The main difference is that the monopolist will set a higher price on the accurately perceived dimension. The higher p_x affects welfare in a subtle way: With utility underestimation and price overestimation, monopoly pricing reduces welfare – the misperception results in inadequately low demand and the high p_x reduces demand even further. With utility overestimation and price underestimation, demand is excessively high in a competitive market. The high p_x set by the monopolist counteracts the inflated demand. But, while the high p_x avoids purchases that generate a social loss, it might also deter purchases that generate a social gain. Accordingly, with utility overestimation and price underestimation the net welfare effect of monopoly pricing is indeterminate.

Now turn to the price-cap itself. At the descriptive level, market power moderates the effect of a price cap on the unregulated price. Consider underestimation – of either utility or price – which could justify a cap on p_y. In a competitive market, a cap that reduces p_y forces the seller to increase p_x, so that the seller covers her overall costs. This unintended consequence – an increase in the unregulated price – is less likely to occur in a monopolistic market. In the pre-cap world, the monopolist may have decided to increase the underestimated price, since the increase did not significantly reduce demand for the monopolist's product. Increasing an accurately perceived price, in response to a cap on the underestimated price, would cost more in terms of reduced demand. Therefore, the monopolist may decide not to increase the unregulated price, or to increase it by a

smaller amount. At the normative level, if the regulated price goes down and the unregulated price does not go up (or not by much), demand will increase. In a monopolistic market, higher demand is welfare-enhancing. In essence, the price cap, in addition to correcting the inefficiency caused by the misperception, can also reduce the monopoly deadweight loss.

F. Related Literature

Markets with multi-dimensional products, and multi-dimensional prices, have been studied in the Industrial Organization (IO) literature. Products with an aftermarket – for parts or service – provide a key example. See Farrell and Klemperer (2007) and Farrell (2008). In the behavioral IO literature, several papers study multi-dimensional pricing. See, e.g., DellaVigna and Malmendier (2004), Gabaix and Laibson (2006), Grubb (2009), Heidhues and Koszegi (2010), Spiegler (2011), Heidhues, Koszegi and Murooka (2012) and Armstrong and Vickers (2012).

These papers by and large do not consider price caps.[1] The important exceptions are DellaVigna and Malmendier (2004), Heidhues and Koszegi (2010) and Armstrong and Vickers (2012). DellaVigna and Malmendier briefly discuss the potential welfare benefits of price regulation. They study naiveté about time preferences, which is related to the utility misperception studied here. This paper extends the analysis in DellaVigna and Malmendier (2004) by considering different types (and directions) of misperception and by comparing the positive and normative implications of the different types (and directions) of misperception. Heidhues and Koszegi (2010) focus on credit contracts, but their model could be generalized. They demonstrate the potential welfare benefits of

[1] Gabaix and Laibson (2006) mention price caps in one short paragraph, with no analysis.

price regulation. Like DellaVigna and Malmendier (2004), Heidhues and Koszegi (2010) study naiveté about time preferences. Heidhues and Koszegi study only the perfect competition case and thus do not identify the effects of market structure on the welfare implications of the price cap.

The model in Armstrong and Vickers (2012) appears to cover both utility and price misperception, but in a way that masks the positive and normative differences between the two types of misperception. Also Armstrong and Vickers study only the perfect competition case. On the other hand, the model in Armstrong and Vickers (2012) is more general than the model in the current paper, since they allow for heterogeneity in consumer misperception (studying markets where some consumers suffer from misperception, but others do not). Armstrong and Vickers show that a price cap can increase welfare, in large part by limiting the cross-subsidization of sophisticated consumers by less sophisticated consumers.

This paper builds on and extends the analysis in Bar-Gill and Bubb (2012). Bar-Gill and Bubb study the effects of a price cap in a simple, linear model where each price, in a two-dimensional pricing scheme, is incurred exactly once. The implication is that prices do not have incentive effects, beyond the purchase decision. The current paper relaxes some of the simplifying assumptions in Bar-Gill and Bubb (2012), generalizing and refining the results of that paper and deriving additional results. Also, while Bar-Gill and Bubb (2012) focus on price underestimation, this paper compares the positive and normative implications of different types (and directions) of misperception. Agarwal et al (2013), in addition to its very sophisticated empirical analysis of the CARD Act, includes a short theory section that uses a model similar to the one developed in Bar-Gill and

Bubb (2012). Like Bar-Gill and Bubb (2012), Agarwal et al (2013) focus on price underestimation. Their treatment of market structure, or market power, is more general than Bar-Gill and Bubb (2012).

G. Roadmap

The framework of analysis is developed in Section 2. The main results are derived in Sections 3-5. Section 6 offers concluding remarks, discussing several extensions: markets where multiple product and price dimensions are subject to misperception; markets where consumers suffer from multiple types of misperception (on a single dimension); indirect forms of price regulation, beyond price-caps; and quality floors in markets where product quality (rather than price) is multi-dimensional. Proofs are relegated to an appendix.

2. Framework of Analysis

A. Basic Setup

Assume a two-dimensional product (X, Y). The consumer chooses how much to consume on each dimension, i.e., the consumer chooses consumption levels (x, y). For simplicity, assume that X is a binary dimension, i.e., $x \in \{0,1\}$, with $x = 1$ representing a decision to purchase the product and $x = 0$ representing a decision not to purchase the product. If the consumer decided to purchase the product, she must then decide how intensely to use the product on the Y dimension, where $y \in R^+$. (The model can be extended to accommodate continuous decisions on both dimensions, e.g., when a consumer decides how much to borrow, on a credit card, in an introductory period (x) and how much to borrow in the post-introductory period (y).)

The assumption is that X and Y are two dimensions of a single product. Or, equivalently, that X and Y are effectively bundled, such that a consumer who purchases X from one seller will not purchase Y from another seller. Moreover, it is assumed that all sellers are offering both X and Y, and that no seller can offer just X (or just Y). The idea is that X and Y are very difficult to separate or, alternatively, that there are substantial efficiencies from bundling them together (or that bundling is very profitable for behavioral reasons).

The seller's cost of providing the product is separable, with an independent per-unit cost for each dimension of the product. There is a fixed cost, c_x, of serving any consumer who chooses to purchase the product, and a per-unit cost, c_y, for each unit of use on dimension Y. The seller's total cost, for a consumer who decided to purchase the product, is: $C(c_x, c_y) = c_x + yc_y$.

The (gross) value of the product to the consumer is: $v + u(y)$, where v is a base-value that is distributed among consumers according to the CDF $F(v)$, and $u(y)$ is a use value that varies with use levels on the Y dimension but in a manner common to all consumers.

B. The Seller's Decisions

The seller sets a two-dimensional price, which is comprised of a per-unit price for each dimension of the product. The per-unit prices are: p_x and p_y. The price p_x will be referred to as the base price; the price p_y will be referred to as the per-use price. The total price is: $P(p_x, p_y) = p_x + yp_y$. The seller's profit per-product purchased is:

$$\pi(p_x, p_y) = P(p_x, p_y) - C(c_x, c_y) = (p_x - c_x) + y(p_y)(p_y - c_y)$$

The seller's total profit function is:

$$\Pi(p_x, p_y) = \pi(p_x, p_y) \cdot D(p_x, p_y)$$

where $D(p_x, p_y)$ represents the demand for the seller's product, i.e., the number of consumers who purchase the product. The demand function is derived below.

The prices that the seller sets, and the profit that the seller makes, depend, among other things, on market structure. I will consider two different assumptions about the structure of the market: perfect competition and monopoly.

C. The Consumer's Decisions

The consumer makes two decisions: (1) whether to purchase the product, and (2) how intensely to use a product that is purchased. I begin by describing the use decision. The prior purchase decision is (potentially) influenced by consumer misperception. I, therefore, present the different types of misperception, before turning to the purchase decision itself.

1. Use Decision

A consumer who has decided to purchase the product will choose a use level, *y*, that solves: $\max_y \langle u(y) - yp_y \rangle$. The First-Order Condition (FOC) is: $u'(y) = p_y$, which implicitly defines the optimal use level as a function of the per-unit price, p_y: $y = y(p_y)$.

2. Consumer Misperception

I consider two different types of misperception: utility misperception, where the consumer believes that her use value will be $\hat{u}(y) = \delta u(y)$, where $\delta \in [0, \infty)$; and price misperception, where the consumer believes that the per-use price will be $\hat{p}_y = \delta p_y$, where $\delta \in [0, \infty)$. I study the two types of misperception separately. (The case where

consumers might suffer from both types of misperception simultaneously is discussed briefly in Section 6.B.). Therefore, I can use the same parameter, δ, for both types of misperception. The benchmark case, where the consumer does not suffer from any misperception, is captured by $\delta = 1$. Underestimation is captured by $\delta < 1$, and overestimation is captured by $\delta > 1$.

Both types of misperception apply only ex ante. Ex post, when the actual use decision is made, the consumer learns her true use value, $u(y)$, and the actual per-use price, p_y, and sets the use level, *y*, accordingly (as described in subsection 1 above). (Compare: naïve hyperbolic discounters in DellaVigna and Malmendier (2004) and Heidhues and Koszegi (2010).) But, ex ante, when making the purchase decision, the consumer thinks that she will choose a different use level:

(1) With utility misperception, the consumer thinks that she will choose a use level, *y*, that solves $\max_y\langle \delta u(y) - yp_y\rangle$. The First-Order Condition (FOC), $\delta u'(y) = p_y$, implicitly defines the anticipated use level as a function of the per-unit price, p_y, and the misperception parameter, δ: $\hat{y} = \hat{y}(p_y; \delta)$.

(2) With price misperception, the consumer thinks that she will choose a use level, *y*, that solves $\max_y\langle u(y) - y\delta p_y\rangle$. The FOC, $u'(y) = \delta p_y$, implicitly defines the anticipated use level as a function of the per-unit price, p_y, and the misperception parameter, δ: $\hat{y} = \hat{y}(p_y; \delta)$.

Note that utility underestimation leads to underestimation of use levels, while price underestimation leads to overestimation of use levels. Conversely, utility overestimation leads to overestimation of use levels, while price overestimation leads to underestimation of use levels.

3. Purchase Decision

The decision whether to purchase the product depends on the (net) value of the product, as perceived by the consumer. The (net) value of the product to a consumer is: $V(v, p_x, p_y) = v + u(y) - (p_x + yp_y)$. This (net) value might be misperceived by the consumer. Specifically, the use dimension – the per-use price, the use level and the use value – are subject to (possible) misperception. The perceived (net) value of the product is: $\hat{V}(v, p_x, p_y; \delta) = v + \hat{u}(\hat{y}) - (p_x + \hat{y}\hat{p}_y)$. This formulation captures the two types of misperception defined in subsection 2 above: With utility misperception, we have $\hat{u}(\hat{y}) = \delta u(\hat{y})$, $\hat{p}_y = p_y$ and $\hat{y} = \hat{y}(p_y; \delta)$; with price misperception, we have $\hat{u}(\hat{y}) = u(\hat{y})$, $\hat{p}_y = \delta p_y$ and $\hat{y} = \hat{y}(p_y; \delta)$.

The consumer will purchase the product iff the perceived (net) value is positive, i.e., iff $\hat{V}(v, p_x, p_y; \delta) > 0$. There exists a threshold value, $\tilde{v}(p_x, p_y; \delta) = (p_x + \hat{y}\hat{p}_y) - \hat{u}(\hat{y})$, such that consumers with $v > \tilde{v}(p_x, p_y; \delta)$ will purchase the product. Assuming a unit mass of consumers, the demand for the product is: $D(p_x, p_y; \delta) = 1 - F\left(\tilde{v}(p_x, p_y; \delta)\right)$.

The perceived overall consumer surplus is:

$$\hat{S}(p_x, p_y; \delta) = \int_{\tilde{v}(p_x, p_y; \delta)}^{\infty} \hat{V}(v, p_x, p_y; \delta) f(v) dv$$

whereas the actual overall consumer surplus is:

$$S(p_x, p_y) = \hat{S}(p_x, p_y; \delta = 1) = \int_{\tilde{v}(p_x, p_y; \delta = 1)}^{\infty} \hat{V}(v, p_x, p_y; \delta = 1) f(v) dv$$

D. Social Welfare

1. The Social Welfare Function

Total social welfare is the sum of utilities enjoyed by consumers who choose to make a purchase, i.e., consumers with $v > \tilde{v}(p_x, p_y; \delta)$, minus the cost – to the seller – of serving these consumers. The social welfare function is:

$$W(p_x, p_y; \delta) = \int_{\tilde{v}(p_x, p_y; \delta)}^{\infty} \left[v + u\left(y(p_y)\right) - \left(c_x + y(p_y)c_y\right) \right] f(v)dv$$

2. The First-Best Optimum

A consumer who decides to purchase the product should choose a use level, *y*, that solves: $\max_y \langle u(y) - yc_y \rangle$. The FOC, $u'(y^*) = c_y$, implicitly defines the optimal use level as: $y^* = y(c_y)$. A consumer should choose to purchase the product *iff* $v > \tilde{v}^* = \tilde{v}(c_x, c_y) = \left(c_x + y(c_y) \cdot c_y\right) - u\left(y(c_y)\right)$. The product should be purchased by the following number of consumers: $D^*(c_x, c_y) = 1 - F\left(\tilde{v}(c_x, c_y)\right)$.

Therefore, social welfare at the first-best optimum is:

$$W^* = W(c_x, c_y; 1) = \int_{\tilde{v}(c_x, c_y)}^{\infty} \left[v + u\left(y(c_y)\right) - \left(c_x + y(c_y)c_y\right) \right] f(v)dv$$

E. The Law

I study the effects of a rule that restricts the permissible magnitude of either p_y or p_x. Specifically, I consider a price cap, $\bar{p}_y$, that adds a "legal constraint" $p_y \leq \bar{p}_y$ to the seller's optimization problem; and a price cap, $\bar{p}_x$, that adds a "legal constraint" $p_x \leq \bar{p}_x$ to the seller's optimization problem. The question is under what conditions will such a law increase social welfare and under what conditions will it decrease social welfare.

The analysis proceeds as follows. I begin, in Section 3, by deriving the second-best optimum that can be attained given a certain level of misperception (δ) – both for utility misperception and for price misperception. Then I derive the equilibrium outcomes and welfare levels and compare them to the second-best benchmark. The analysis depends on market structure. The perfect competition case is considered in Section 4. The monopoly case is considered in Section 5.

3. The Second-Best Optimum

A. General

The benchmark for the welfare analysis is the second-best optimum, namely, the maximum welfare level that can be attained given a certain level of misperception (δ). This second-best optimum is attained by setting prices, $p_y^*(\delta)$ and $p_x^*(\delta)$, to maximize the social welfare function (1), s.t. the seller's participation constraint: $\Pi\left(p_x, p_y; \delta\right) \geq 0$. Solving this maximization problem, we can characterize the second-best prices and the welfare implications of consumer misperception and of the socially optimal pricing response to misperception.

B. Utility Misperception

I begin with utility misperception. The results are summarized in Lemma 1.

Lemma 1 (Second-Best Optimum, Utility Misperception):

(a) When use value is underestimated, i.e., when $\delta < 1$*: The second-best per-use price,* p_y*, satisfies:* $p_y^*(\delta) > c_y$*, and the second-best base price,* p_x*, satisfies:* $p_x^*(\delta) < c_x$*. Social welfare is reduced by the misperception. Given the misperception, optimal pricing balances the social cost of distorted use levels (generated by* $p_y^*(\delta) > c_y$*) against the social benefit from reducing distortions in purchase levels.*

(b) When use value is overestimated, i.e., when $\delta > 1$*: The second-best per-use price,* p_y*, satisfies:* $p_y^*(\delta) > c_y$*, and the second-best base price,* p_x*, satisfies:* $p_x^*(\delta) < c_x$*. Social welfare is reduced by the misperception. Given the misperception, optimal pricing balances the social cost of distorted use levels (generated by* $p_y^*(\delta) > c_y$*) against the social benefit from reducing distortions in purchase levels.*

Remark:

Given the misperception, cost-based pricing, i.e., $p_y = c_y$ and $p_x = c_x$, is no longer optimal. Rather it is optimal to increase p_y (above c_y) and reduce p_x (below c_x). This pricing pattern holds for both under- and overestimation of use values, but for different reasons:

(a) Underestimation ($\delta < 1$): Consider an increase in p_y (above c_y). This increase has two effects: (1) it decreases the actual value of the product, as the higher per-use price leads to inefficiently low use levels; and (2) it increases the perceived value of the product, since the increase in p_y and decrease in p_x counteract the reduction in demand caused by the misperception. Because of Effect (1), a higher p_y (and lower p_x) is bad for the infra-marginal consumers who would have purchased even when

$p_y = c_y$ (and $p_x = c_x$). Because of Effect (2), a higher p_y (and lower p_x) is good for the marginal consumers who now purchase and gain positive actual value. Since Effect (1) is zero when $p_y = c_y$ (and $p_x = c_x$), the second-best prices satisfy: $p_y^*(\delta) > c_y$ and $p_x^*(\delta) < c_x$.

(b) Overestimation ($\delta > 1$): The difference between over- and underestimation is in Effect (2). With overestimation, the first-order effect of the misperception is to artificially inflate demand. Second-best optimal pricing thus works to reduce demand. When use is overestimated, and thus the effect of p_y is overestimated, an increase in p_y, and decrease in p_x, achieves this demand-reduction goal.

C. Price Misperception

Next consider price misperception. The results are summarized in Lemma 2.

Lemma 2 (Second-Best Optimum, Price Misperception):

(a) When the per-use price is underestimated, i.e., when $\delta < 1$: The second-best per-use price, p_y, satisfies: $p_y^*(\delta) < c_y$, and the second-best base price, p_x, satisfies: $p_x^*(\delta) > c_x$. Social welfare is reduced by the misperception. Given the misperception, optimal pricing balances the social cost of distorted use levels (generated by $p_y^*(\delta) < c_y$) against the social benefit from reducing distortions in purchase levels.

(b) When the per-use price is overestimated, i.e., when $\delta > 1$: The second-best per-use price, p_y, satisfies: $p_y^*(\delta) < c_y$, and the second-best base price, p_x, satisfies: $p_x^*(\delta) > c_x$. Social welfare is reduced by the misperception. Given the misperception, optimal

pricing balances the social cost of distorted use levels (generated by $p_y^*(\delta) < c_y$) against the social benefit from reducing distortions in purchase levels.

Remark:

With price misperception, as with utility misperception, cost-based pricing, i.e., $p_y = c_y$ and $p_x = c_x$, is no longer optimal. The second-best optimal prices, however, are very different with price misperception. Specifically, it is optimal to reduce p_y (below c_y) and increase p_x (below c_x). This pricing pattern holds for both under- and overestimation of the per-use price:

(a) Underestimation ($\delta < 1$): While utility underestimation results in inadequately low demand, price underestimation results in excessive demand. To counter-act this first-order effect of the misperception, the second-best optimum requires a decrease in the misperceived price, p_y, and a corresponding increase in the accurately perceived price, p_x. This price adjustment reduces the adverse effect of the misperception.

(b) Overestimation ($\delta > 1$): While utility overestimation results in excessive demand, price overestimation results in inadequately low demand. To counter-act this first-order effect of the misperception, the second-best optimum requires a decrease in the misperceived price, p_y, and a corresponding increase in the accurately perceived price, p_x.

With both under- and overestimation, it is second-best optimal to shift pricing away from the misperceived price dimension and to the accurately perceived price dimension.

D. Comparison

The second-best optimum pricing structure critically depends on the type of misperception. With utility misperception, the optimal response to both under- and overestimation entails a high per-use price and a low base price. Conversely, with price misperception, the optimal response to both under- and overestimation entails a low per-use price and a high base price.

4. Competition

A. General

In a competitive market, sellers set prices to maximize the perceived consumer surplus, subject to a break-even constraint. Formally, the seller solves the following maximization problem:

$$\max_{p_x,p_y} \hat{S}(p_x,p_y;\delta) \text{ s.t. } \Pi(p_x,p_y)=0$$

s.t. $\Pi(p_x,p_y)=0$. The second-best optimum (see Section 3 above) was derived by maximizing

$$W(p_x,p_y;\delta)=\int_{\tilde{v}(p_x,p_y;\delta)}^{\infty}\left[v+u\left(y(p_y)\right)-\left(c_x+y(p_y)c_y\right)\right]f(v)dv$$

s.t. $\Pi(p_x,p_y)=0$. Sellers in a competitive market care about maximizing the perceived value to their customers, not about maximizing actual surplus. This is the source of the inefficiency.

Note that misperception is the underlying problem. Without misperception, i.e., when $\delta=1$, the per-use price, p_y, is: $p_y^C=c_y$, the base price, p_x, is: $p_x^C=c_x$, and the first-best socially optimal welfare level is obtained. These are simply the standard results regarding

the efficiency properties of perfect competition, under the (standard) assumption that consumers do not suffer from any misperception. I next study the effects of consumer misperception.

B. Utility Misperception

Lemma 3 describes equilibrium outcomes in a competitive market with utility misperception and evaluates their welfare implications.

Lemma 3 (Competitive Equilibrium, Utility Misperception): In a competitive market –

(a) When use value is underestimated, i.e., when $\delta < 1$*: The per-use price,* p_y*, satisfies:* $p_y^C(\delta) > p_y^*(\delta) > c_y$*, and the base price,* p_x*, satisfies:* $p_x^C(\delta) < p_x^*(\delta) < c_x$*. Social welfare is reduced by the misperception. Given the misperception, equilibrium prices deviate from the second-best optimal prices, distorting use levels and inducing excessive demand.*

(b) When use value is overestimated, i.e., when $\delta > 1$*: The per-use price,* p_y*, satisfies:* $p_y^C(\delta) < c_y < p_y^*(\delta)$*, and the base price,* p_x*, satisfies:* $p_x^C(\delta) > c_x > p_x^*(\delta)$*. Social welfare is reduced by the misperception. Given the misperception, equilibrium prices deviate from the second-best optimal prices, indeed they move in the wrong direction vis-à-vis cost, distorting use levels and inducing excessive demand.*

Remark:

(a) Utility Underestimation:

The per-use price. If consumers underestimate use-values, they will also underestimate use levels and the importance of the per-use price, p_y. Sellers, responding

to this misperception, will increase p_y above the efficient level. Second-best optimal price adjustment, in response to the misperception, results in $p_y^*(\delta) > c_y$ and $p_x^*(\delta) < c_x$ (see Lemma 1). These second-best prices balance the two effects of an increase in p_y (above c_y): (1) A decrease in the actual value of the product, as the higher per-use price leads to inefficiently low use levels; and (2) An increase in the perceived value of the product, as the increase in p_y and decrease in p_x counteract the reduction in demand caused by the misperception. Sellers care only about increasing demand (Effect (1)) and not about the actual value provided to consumers (Effect (2)). Therefore, equilibrium prices will deviate from the second-best prices. Specifically, p_y will be set too high.

The base price. The zero-profit condition implies that if p_y is set above cost, such that sellers are making positive profits on the use dimension, then p_x must be set below cost. Sellers increase p_y to extract more revenue from the use dimension, since this allows them to extract money from consumers while incurring a smaller cost in terms of lost demand (because of the misperception). In a competitive market, if sellers extract more revenue on the use dimension (the Y dimension), they must extract less through the base price, or incur a bigger loss on the X dimension. Therefore, an increase in p_y above the second-best optimal level will be accompanied by a decrease in p_x below the second-best optimal level.

Welfare. The excessively high p_y will distort use-level decisions (beyond what is optimal, given the misperception). In addition, the excessively high p_y, and corresponding p_x, will inefficiently inflate demand, such that the marginal gain from more purchases is outweighed by the infra-marginal loss in the value of each purchase.

(b) Utility Overestimation: If consumers overestimate use-values, they will also overestimate use levels and the importance of the per-use price, p_y. Sellers, responding to this misperception, will decrease p_y below c_y (Recall, from Lemma 1, that second-best efficiency requires p_y above c_y.) When $p_y < c_y$, the zero-profit condition implies $p_x > c_x$. (Recall, from Lemma 1, that second-best efficiency requires p_x below c_x.) With utility overestimation, equilibrium prices go in opposite direction, vis-à-vis cost, from the second-best prices. While second-best prices work to mitigate the effect of the misperception, equilibrium prices exacerbate the effect of the misperception. The result is inefficiently inflated demand. In addition, the deviations from cost-based pricing distorts use-level decisions.

We can now study the effects of imposing a price cap $\bar{p}_y$ or $\bar{p}_x$, namely of adding a legal constraint $p_y \leq \bar{p}_y$ or $p_x \leq \bar{p}_x$. We first note the standard result that, without misperception, a price cap can only reduce social welfare. In our model, it is the misperception that generates the welfare costs and opens the door for potentially welfare-enhancing regulation. In particular, when consumers underestimate the use value, i.e., when $\delta < 1$, the per-use price, p_y, will be excessively high without legal intervention (see Lemma 3), and so a price cap, $\bar{p}_y$, can increase social welfare (as long as the cap is not set too low). When consumers overestimate the use value, i.e., when $\delta > 1$, the base price, p_x, will be excessively high without legal intervention (see Lemma 3), and so a properly calibrated price cap, $\bar{p}_x$, can increase social welfare.

These results are summarized in Proposition 1.

Proposition 1 (Competition, Utility Misperception): In a competitive market –

(a) When use value is underestimated, i.e., when $\delta < 1$, a mild, though still binding, price cap, $\bar{p}_y$, satisfying $p_y^C(\delta) > \bar{p}_y \geq p_y^*(\delta) > c_y$, reduces the per-use price, p_y, and increases the base price, p_x, resulting in increased social welfare.

(b) When use value is overestimated, i.e., when $\delta > 1$, a mild, though still binding, price cap, $\bar{p}_x$, satisfying $p_x^C(\delta) > \bar{p}_x \geq p_x^*(\delta)$, reduces the base price, p_x, and increases the per-use price, p_y, resulting in increased social welfare.

Remark: The results stated in Proposition 1 follow from Lemma 3.

(a) Utility Underestimation: A mild, yet binding legal constraint brings the per-use price closer to the second-best optimal price, increasing social welfare. The reduction in p_y is accompanied by a corresponding increase in p_x. Note, however, that a strict constraint, $\bar{p}_y < p_y^*(\delta)$, can reduce welfare. A strict constraint reduces the per-use price below the second-best optimal price. The base price, p_x, increases in response. This can either increase or decrease social welfare, as we move from an excessively high price to an inadequately low price.

(b) Utility Overestimation: The relevant cap here is on the base price, p_x. The second-best optimal base price, $p_x^*(\delta)$, is below cost, while the equilibrium base price (without legal intervention) is above cost. Accordingly, there is more room for a welfare-enhancing price cap. And, still, the lawmaker should take care not to set $\bar{p}_x < p_x^*(\delta)$, since such a strict cap might reduce welfare.

C. Price Misperception

Lemma 4 describes equilibrium outcomes in a competitive market with price misperception and evaluates their welfare implications.

Lemma 4 (Competitive Equilibrium, Price Misperception): In a competitive market –

(a) When the per-use price is underestimated, i.e., when $\delta < 1$*: The per-use price,* p_y*, satisfies:* $p_y^C(\delta) > c_y > p_y^*(\delta)$*, and the base price,* p_x*, satisfies:* $p_x^C(\delta) < c_x < p_x^*(\delta)$*. Social welfare is reduced by the misperception. Given the misperception, equilibrium prices deviate from the second-best optimal prices, indeed they move in the wrong direction vis-à-vis cost, distorting use levels and inducing excessive demand.*

(b) When the per-use price is overestimated, i.e., when $\delta > 1$*: The per-use price,* p_y*, satisfies:* $p_y^C(\delta) < p_y^*(\delta) < c_y$*, and the base price,* p_x*, satisfies:* $p_x^C(\delta) > p_x^*(\delta) > c_x$*. Social welfare is reduced by the misperception. Given the misperception, equilibrium prices deviate from the second-best optimal prices, indeed they move in the wrong direction vis-à-vis cost, distorting use levels and inducing excessive demand.*

Remark:

(a) Price Underestimation: When the per-use price is underestimated, sellers increase this price dimension (and reduce the base price) to exacerbate the effect of the misperception and further inflate demand.

(b) Price Overestimation: When the per-use price is overestimated, sellers shift pricing away from this price dimension, and towards the base price. Here the misperception reduces demand, and the price shift minimizes this demand-reducing effect.

We can now study the effects of imposing a price cap. With (price) underestimation, i.e., when $\delta < 1$, the per-use price, p_y, will be excessively high without legal intervention (see Lemma 4), and so a price cap, $\bar{p}_y$, can increase social welfare. With (price) overestimation, i.e., when $\delta > 1$, the base price, p_x, will be excessively high without legal intervention (see Lemma 4), and so a price cap, $\bar{p}_x$, can increase social welfare.

These results are summarized in Proposition 2.

Proposition 2 (Competition, Price Misperception): In a competitive market –

(a) When the per-use price is underestimated, i.e., when $\delta < 1$, a mild, though still binding, price cap, $\bar{p}_y$, satisfying $p_y^C(\delta) > \bar{p}_y \geq p_y^*(\delta)$, reduces the per-use price, p_y, and increases the base price, p_x, resulting in increased social welfare.

(b) When use value is overestimated, i.e., when $\delta > 1$, a mild, though still binding, price cap, $\bar{p}_x$, satisfying $p_x^C(\delta) > \bar{p}_x \geq p_x^*(\delta) > c_x$, reduces the base price, p_x, and increases the per-use price, p_y, resulting in increased social welfare.

Remark: The results stated in Proposition 2 follow from Lemma 4.

(a) Price Underestimation: The second-best optimal per-use price, $p_y^*(\delta)$, is below cost, while the equilibrium per-use price (without legal intervention) is above cost. Accordingly, there is more room for a welfare-enhancing price cap.

(b) Price Overestimation: A mild, yet binding legal constraint brings the base price closer to the second-best optimal price, increasing social welfare.

Remark (Political Economy): Policymakers intervene and cap a price when this price dimension, and its adverse implications, become politically salient. If a price dimension that has become politically salient has also become salient to consumers, i.e., the misperception has been cured, then legal intervention will only harm consumers and reduce welfare. This link between political salience and market salience is more likely with price misperception (Proposition 2), but less likely with utility misperception (Proposition 1).

D. Comparison: The Object (and Direction) of Misperception

To facilitate a comparison between the welfare and policy implications of different types (and directions) of misperception, we collect the results from Lemmas 3 and 4 in the following Table:

	Utility Misperception	Price Misperception
Underestimation	$p_y^C(\delta) > p_y^*(\delta) > c_y$ $p_x^C(\delta) < p_x^*(\delta) < c_x$	$p_y^C(\delta) > c_y > p_y^*(\delta)$ $p_x^C(\delta) < c_x < p_x^*(\delta)$
Overestimation	$p_y^C(\delta) < c_y < p_y^*(\delta)$ $p_x^C(\delta) > c_x > p_x^*(\delta)$	$p_y^C(\delta) < p_y^*(\delta) < c_y$ $p_x^C(\delta) > p_x^*(\delta) > c_x$

Table 1: Price Distortions for Different Types (and Directions) of Misperception

Underestimation. Equilibrium per-use prices are similarly above cost with both utility and price misperception, but the second-best prices are different and, accordingly, the welfare implications are different: With utility misperception, the second-best price is above the per-use cost, c_y, and the equilibrium price is even higher: $p_y^C(\delta) > p_y^*(\delta) > c_y$. With price misperception, the second-best price is below the per-use cost, c_y, and the equilibrium price is above the per-use cost: $p_y^C(\delta) > c_y > p_y^*(\delta)$. Accordingly, the welfare cost is larger with price misperception. (While $p_y^C(\delta) > c_y$ for both utility and price misperception, the exact price depends on the type of misperception.)

The optimal scope of legal intervention depends on the welfare cost. A price cap, $\bar{p}_y$, in the range $\left[p_y^*(\delta), p_y^C(\delta)\right]$ increases welfare with certainty. This range, we have now seen, is larger with price misperception. Specifically, with utility misperception a price cap above the per-unit cost, c_y, might fall outside the range, and thus reduce welfare; with price misperception a price cap above the per-unit cost, c_y, will always increase welfare.

Overestimation. With overestimation, we focus on the base-price, rather than on the per-use price. The welfare picture is the mirror image of what we saw for underestimation: The welfare cost, and the scope for welfare-enhancing price caps, are larger with utility misperception.

Summary. Market outcomes are often sensitive to the type of misperception that consumers suffer from. Here, however, we find that two types of plausible misperceptions – utility misperception and price misperception – have very similar effects on equilibrium prices. Nevertheless, since second-best optimal pricing depends on

the type of misperception, welfare and policy implications also depend on the type of misperception.

These comparisons are depicted graphically in Figure 1 below.

[Figure 1 goes here. The Figure is provided at the end of this document.]

5. Monopoly

A. General

We next consider a monopolistic market. A monopolistic seller sets prices, p_x and p_y, to maximize its profit function: $\Pi(p_x, p_y) = \pi(p_x, p_y) \cdot \left[1 - F\left(\tilde{v}(p_x, p_y; \delta)\right)\right]$.

This model can be used to replicate standard monopoly results, under the (standard) assumption that consumers do not suffer from any misperception. The monopolist would set an efficient per-use price, $p_y^M = c_y$, to maximize total surplus (a different p_y distorts use-level decisions and reduces total surplus) and use the base price to extract monopolistic rents. The base price would be set above the competitive level, which leads to an inefficiently small number of products purchased. The result is a welfare loss – the monopoly deadweight loss. Our focus, however, is on the implications – both descriptive and normative – of consumer misperception. We begin with utility misperception, in subsection B and then turn to price misperception in subsection C. Subsection D compares the results across the different types (and directions) of misperception and highlights the implications of market power.

B. Utility Misperception

Lemma 5 describes equilibrium outcomes in a monopolistic market with utility misperception and evaluates their welfare implications.

Lemma 5 (Monopolistic Market, Utility Misperception): In a monopolistic market –

(a) When use value is underestimated, i.e., when $\delta < 1$*:*

i. The per-use price, p_y*, satisfies:* $p_y^M(\delta) > p_y^*(\delta) > c_y$.

ii. The base price, p_x*, can be smaller than, equal to, or greater than* c_x*. The base price can be smaller than, equal to, or greater than* $p_x^*(\delta)$.

iii. Social welfare is reduced by the misperception. Since $p_y^M(\delta) = p_y^C(\delta)$*, the welfare cost can be divided into two components: (1) the cost induced by the misperception (which is identical to the welfare cost under Competition):* $W\left(p_y^*(\delta), p_x^*(\delta)\right) - W\left(p_y^M(\delta) = p_y^C(\delta), p_x^C(\delta)\right)$*; and (2) the cost induced by the monopoly deadweight loss:* $W\left(p_y^M(\delta) = p_y^C(\delta), p_x^C(\delta)\right) - W\left(p_y^M(\delta), p_x^M(\delta)\right)$.

(b) When use value is overestimated, i.e., when $\delta > 1$*:*

i. The per-use price, p_y*, satisfies:* $p_y^M(\delta) < c_y < p_y^*(\delta)$.

ii. The base price, p_x*, satisfies:* $p_x^M(\delta) > c_x > p_x^*(\delta)$.

iii. Social welfare is reduced by the misperception. Since $p_y^M(\delta) = p_y^C(\delta)$*, the welfare cost can be divided into two components: (1) the cost induced by the misperception (which is identical to the welfare cost under Competition):* $W\left(p_y^*(\delta), p_x^*(\delta)\right) - W\left(p_y^M(\delta) = p_y^C(\delta), p_x^C(\delta)\right)$*; and (2) the welfare effect of*

the higher monopoly base price, $W\left(p_y^M(\delta) = p_y^C(\delta), p_x^C(\delta)\right) - W\left(p_y^M(\delta), p_x^M(\delta)\right)$ *, which can be either positive or negative.*

Remark:

(a) Utility Underestimation:

The per-use price. As in the perfect competition case, misperception leads to an inefficiently high per-use price, p_y. In the absence of misperception, the monopolist maximizes surplus by setting $p_y = c_y$ and then balances higher per-unit profits against reduced demand by calibrating the base price, p_x. Misperception hurts the monopolist's bottom line by reducing demand. The monopolist responds by shifting its pricing from p_x to the underestimated p_y – to minimize the reduction in demand. The equilibrium pricing scheme trades-off two countervailing effects on demand: (1) a higher per-unit price ($p_y > c_y$) distorts use levels and thus reduces both the per-product surplus and, correspondingly, demand for the product (albeit in a way that is partly muted by the misperception), and (2) a shift away from the accurately perceived p_x to the underestimated p_y increases demand.

The existence of an underestimated price dimension mutes the standard monopoly trade-off between (1) the infra-marginal benefit of a higher price, a larger profit per-product sold, and (2) the marginal cost of a higher price, a reduction in the number of products sold. The misperception reduces the marginal cost when the monopolist shifts pricing to the underestimated dimension.

The base price. In the standard, one-price model, the monopolist trades-off the infra-marginal benefit of a higher price (a larger profit per-product sold) against the marginal cost of a higher price (a reduction in the number of products sold). In the present two-price model, this trade-off applies, in a rough sense, to the total price. Since the monopolist responds to consumer misperception by increasing p_y, it needs to reduce p_x in order to rebalance the benefit-cost tradeoff.

Welfare. The welfare costs under Monopoly are equal to the welfare costs under Competition, plus the monopoly deadweight loss. Note that, given the misperception, demand is too high under Competition, and the higher p_x under Monopoly reduces demand. But this reduction does not compensate for the welfare loss that the high p_y caused (recall that $p_y^M = p_y^C$). The higher p_x only adds a monopoly deadweight loss.

(b) Utility Overestimation: As in the perfect competition case, misperception leads to an inefficiently low per-use price, p_y. Moreover, whereas the second-best per-use price is above cost, the equilibrium price is below cost. The monopolist compensates for the low per-use price, which implies a loss on the use dimension, with a higher base price. The monopoly base price is higher than the competitive base price, and it reduces demand below the competitive level. This, however, can be a good thing, because, with overestimation, at least some of these lost purchases would have generated a net social loss.

We can now study the effects of imposing a price cap. Begin with the no misperception case and consider, first, a cap $\bar{p}_y$ on the per-use price. A price cap $\bar{p}_y \geq c_y$ has no effect, since the constraint is not binding. A price cap, $\bar{p}_y < c_y$, distorts prices and

reduces welfare. Specifically, the reduced p_y leads to excessive use-levels. The monopolist will increase the base price, p_x, to compensate for losses on the use dimension. Next, consider a cap $\bar{p}_x$ on the base price. Note that in a standard monopoly model, with a single price dimension, a price cap can reduce the monopoly deadweight loss and thus increase welfare. In our two-dimensional price model, a cap on the base price can similarly reduce the monopoly deadweight loss. But the cap will also result in a use-level distortion, as the monopolist increases p_y in response to the cap on p_x. The overall welfare effect of a cap on p_x is, therefore, ambiguous.

Our focus, however, is on the effect of a price cap, given consumer misperception. Without legal intervention, underestimation of use value, i.e., $\delta < 1$, results in an excessively high p_y and in a correspondingly low use-level. This distortion leads to a welfare loss. A price cap, $\bar{p}_y$, will reduce the use-level distortion (as long as the cap is not set too low). The effects of a price cap, $\bar{p}_y$, on the number of products purchased depends on how a price cap on p_y affects the monopolist's choice of p_x. The monopolist realizes that the effect, on demand, of a higher p_x, is larger than the effect of a higher p_y, which is muted by the misperception. Accordingly, the monopolist will hesitate to compensate for the lower p_y by increasing p_x. As a result, the price cap can also reduce the monopoly deadweight loss. In sum, a price cap can increase overall welfare. When consumers overestimate the use value, i.e., when $\delta > 1$, it is the base price that is set too high, and so a cap on p_x can increase welfare, if properly calibrated.

These results are summarized in Proposition 3.

Proposition 3 (Monopoly, Utility Misperception): In a monopolistic market –

(a) When use value is underestimated, i.e., when $\delta < 1$, a mild, though still binding, price cap, $\bar{p}_y$, satisfying $p_y^M(\delta) > \bar{p}_y \geq p_y^*(\delta) > c_y$, reduces the per-use price, p_y, and increases the base price, p_x, resulting in increased social welfare.

(b) When use value is overestimated, i.e., when $\delta > 1$, a mild, though still binding, price cap, $\bar{p}_x$, satisfying $p_x^M(\delta) > \bar{p}_x \geq p_x^*(\delta)$, reduces the base price, p_x, and increases the per-use price, p_y, resulting in increased social welfare.

Remark: The results stated in Proposition 3 follow from Lemma 5. These results, for a monopolistic market, largely mirror the results stated in Proposition 1, for a competitive market.

C. Price Misperception

Lemma 6 describes equilibrium outcomes in a monopolistic market with price misperception and evaluates their welfare implications.

Lemma 6 (Monopolistic Market, Price Misperception): In a monopolistic market –

(a) When the per-use price is underestimated, i.e., when $\delta < 1$:

i. The per-use price, p_y, satisfies: $p_y^M(\delta) > c_y > p_y^*(\delta)$.

ii. The base price, p_x, can be smaller than, equal to, or greater than c_x. The base price can be smaller than, equal to, or greater than $p_x^*(\delta)$.

iii. Social welfare is reduced by the misperception. Since $p_y^M(\delta) = p_y^C(\delta)$, the welfare cost can be divided into two components: (1) the cost induced by the

misperception (which is identical to the welfare cost under Competition): $W\left(p_y^*(\delta), p_x^*(\delta)\right) - W\left(p_y^M(\delta) = p_y^C(\delta), p_x^C(\delta)\right)$*; and (2) the welfare effect of the higher monopoly base price,* $W\left(p_y^M(\delta) = p_y^C(\delta), p_x^C(\delta)\right) - W\left(p_y^M(\delta), p_x^M(\delta)\right)$*, which can be either positive or negative.*

(b) When use value is overestimated, i.e., when $\delta > 1$*:*

i. The per-use price, p_y*, satisfies:* $p_y^M(\delta) < p_y^*(\delta) < c_y$.

ii. The base price, p_x*, satisfies:* $p_x^M(\delta) > p_x^*(\delta) > c_x$.

iii. Social welfare is reduced by the misperception. Since $p_y^M(\delta) = p_y^C(\delta)$*, the welfare cost can be divided into two components: (1) the cost induced by the misperception (which is identical to the welfare cost under Competition):* $W\left(p_y^*(\delta), p_x^*(\delta)\right) - W\left(p_y^M(\delta) = p_y^C(\delta), p_x^C(\delta)\right)$*; and (2) the cost induced by the monopoly deadweight loss:* $W\left(p_y^M(\delta) = p_y^C(\delta), p_x^C(\delta)\right) - W\left(p_y^M(\delta), p_x^M(\delta)\right)$.

Remark:

As in the competition case, the welfare cost is larger with price underestimation, as compared to utility underestimation. And, conversely, the welfare cost is smaller with price overestimation, as compared to utility overestimation.

As with utility misperception, the monopoly base price is higher than the competitive base price, and it reduces demand below the competitive level. When price is overestimated, this demand reduction effect reduces welfare. However, when price is

underestimated, the reduced demand can increase welfare, since at least some of these lost purchases would have generated a net social loss. (With utility misperception, we obtained the opposite results, namely, that the lower demand is welfare reducing for utility underestimation, but can be welfare enhancing for utility overestimation.)

We can now study the effects of imposing a price cap. These effects are summarized in Proposition 4.

Proposition 4 (Monopoly, Price Misperception): In a monopolistic market –

(a) When the per-use price is underestimated, i.e., when $\delta < 1$, a mild, though still binding, price cap, $\bar{p}_y$, satisfying $p_y^M(\delta) > \bar{p}_y \geq p_y^*(\delta)$, reduces the per-use price, p_y, and increases the base price, p_x, resulting in increased social welfare.

(b) When the per-use price is overestimated, i.e., when $\delta > 1$, a mild, though still binding, price cap, $\bar{p}_x$, satisfying $p_x^M(\delta) > \bar{p}_x \geq p_x^*(\delta) > c_x$, reduces the base price, p_x, and increases the per-use price, p_y, resulting in increased social welfare.

Remark: The results stated in Proposition 4 follow from Lemma 6. These results, for a monopolistic market, largely mirror the results stated in Proposition 3, for a competitive market.

D. Comparison: The Object (and Direction) of Misperception and Market Structure

I have shown that equilibrium per-use prices are independent of market structure. Accordingly, the comparison conducted in Section 4.D. holds also in the monopoly case:

the welfare loss, and the scope for welfare-enhancing price caps (on the p_y dimension), is greater with price underestimation than with utility underestimation. And, conversely, the welfare loss, and the scope for welfare-enhancing price caps (on the p_x dimension), is greater with utility overestimation than with price overestimation.

The effect of market structure manifests in the higher base price. Utility underestimation and price overestimation reduce the difference between p_x^M and p_x^C, since these misperceptions reduce demand and thus reduce the benefit to the monopolist from raising the base price. In contrast, utility overestimation and price underestimation increase the difference between p_x^M and p_x^C, since these misperceptions increase demand and thus increase the benefit to the monopolist from raising the base price.

What are the welfare implications of the higher base price that the monopolist sets? Since $p_y^M(\delta) = p_y^C(\delta)$, we have:

$$W\left(p_x^C(\delta), p_y^C(\delta); \delta\right) - W\left(p_x^M(\delta), p_y^M(\delta); \delta\right) =$$

$$= \int_{\tilde{v}(p_y, p_x^C; \delta)}^{\tilde{v}(p_y, p_x^M; \delta)} \left[\left(v + u\left(y(p_y)\right) - c_x - y(p_y)c_y\right)\right] f(v)dv$$

This difference is positive with utility underestimation and price overestimation – monopoly pricing reduces welfare. With utility overestimation and price underestimation, the difference can be either positive or negative. The higher base price reduces demand. The reduced demand avoids purchases that generate a social loss, but it might also deter purchases that generate a social gain. Accordingly, with utility overestimation and price underestimation the net welfare effect of monopoly pricing is indeterminate. (See Lemma 5 and Lemma 6.)

Finally, I consider how the positive and normative implications of a price cap depend on market structure. I focus on the case of underestimation – both utility underestimation and price underestimation, where the price cap is imposed on the per-use price. This comparison is meaningful, since the pre-cap price is independent of market structure. (This is not true for the base price, which would be capped in the case of overestimation.) I begin by asking how a cap on the per-use price affects the base price and, specifically, how this effect differs between Monopoly and Competition. A decrease in the per-use price (because of the cap) reduces the seller's revenue form the use dimension. In a competitive market, sellers will have to increase the base price to compensate for this shortfall in revenues. The same is not true in a monopolistic market: With a mild price cap, the monopolist may decide *not* to increase p_x in response to the reduction in p_y. Before increasing a price, the monopolist considers the detrimental effect of the price increase on demand for its products. For the per-use price, p_y, this detrimental effect is moderated by the misperception (the consumer underestimates the per-use price itself or underestimates the use-level and thus the total use-based price). No such moderation exists for the base price, p_x. Therefore, the monopolist may decide to absorb the reduction in profit imposed by the price cap, or some of it, rather than to try and compensate by increasing the base price.

I can now state the following proposition.

Proposition 5 (Underestimation, Effects of a Cap on the Unregulated Price):

With both utility underestimation and price underestimation, the increase in the base price, p_x, as a result of a price cap, $\bar{p}_y$, will be smaller in a monopolistic

market, as compared to a competitive market. The difference is increasing in the magnitude of the misperception.

Next consider the welfare implications of the price cap and how they depend on market structure. Since the per-use price does not depend on market structure (neither pre-cap nor post-cap), the welfare effects of market structure are determined by the effects on the base price (see Proposition 5 above). When the monopolist only minimally adjust the base price, in response to a price cap, $\bar{p}_y$, the price cap reduces the monopoly deadweight loss, in addition to correcting the distortion caused by the seller's response to the misperception.

6. Concluding Remarks

A. Misperception on Multiple Dimensions

The model studied in this paper allowed for misperception with respect to one product dimension (the Y dimension) – either misperception about the utility derived from this dimension or about the price associated with this dimension. The other product dimension (the X dimension) was assumed to be free of misperception. What happens when both dimensions are subject to misperception?

While the model in this paper studies one misperceived dimension and one accurately perceived dimension, the analysis could be readily extended to the case where both dimensions are subject to misperception. The dimension where misperception is more severe would be the Y dimension and the dimension where misperception is less severe would be the X dimension.

The analysis, and results, would need to be reconsidered, when the number of dimensions, and specifically the number of dimensions subject to misperception, is greater than two. Consider underestimation – of utility or of price – where a price cap on the misperceived dimension was shown to increase social welfare. The benefit from imposing a price cap would be reduced, if the seller can easily find a third dimension, where the misperception is of nearly equal magnitude.

B. Multiple Misperceptions on a Single Dimension

The analysis in this paper studied the effects of each misperception – utility misperception (under- and over-estimation) and price misperception (under- and over-estimation) – separately. What happens if consumers suffer from multiple misperceptions simultaneously? A full analysis of this richer model is beyond the scope of this paper. But even without a full analysis it is clear that simultaneous misperceptions can either reinforce one another or offset one another in non-trivial ways.

For example, utility underestimation and price underestimation both push the pre-cap, per-use price, p_y, up above cost. These same misperceptions, however, exert offsetting forces on the second-best price: utility underestimation pushes the second-best price up above cost, whereas price underestimation pulls the second-best price down below cost. The policy implications are, thus, indeterminate. For example, assume that consumers are known to underestimate the per-use price in a certain market, and the lawmaker now learns that utility is also underestimated. This discovery could either strengthen the case for imposing a price cap, because the utility underestimation further

increases the pre-cap price, or it could weaken the case for imposing a price cap, because the additional misperception increases the second-best price.

A different set of interactions occurs if we consider utility underestimation and price overestimation. These two misperceptions exert offsetting forces on the pre-cap, per-use price: utility underestimation pushes p_y up above cost, whereas price overestimation pulls the pre-cap price down below cost. With opposite effects on p_x. They also exert offsetting forces on the second-best price: utility underestimation pushes the second-best per-use price up above cost, whereas price overestimation pulls the second-best per-use price down, below cost. With opposite effects on the second-best base price. Once again, the policy implications are ambiguous.

C. Beyond Price-Caps

The analysis in this paper and the policy implications that follow from it may apply beyond price-caps, to other, indirect forms of price regulation. Policymakers can, and do, restrict prices in other ways. For example, the CARD Act restricts sellers' ability to reprice credit card debt based on new information regarding the probability that the cardholder will default. Lawmakers reduce prices by changing defaults and demanding that con sumers explicitly opt-into the targeted service (as with credit card overlimit fees and overdraft protection). Finally, policymakers can influence pricing by mandating conspicuous disclosure of a specific price dimension (e.g., large font, Bold face terms in the standardized credit card disclosure, the Schumer Box) or including a certain price dimension in an influential aggregate disclosure (e.g., specifying what fees are included in the "finance charge" definition, which underlies the APR disclosure). If these

disclosure strategies succeed in focusing competition on the targeted price dimension, the result would be downward pressure on the regulated price dimension.

Like price caps, these alternative price-control policies often target one price dimension within a multi-dimensional pricing structure. While each policy has unique features and merits further study, the analysis in this paper should be informative – in terms of both market outcomes and social welfare.

D. Multi-Dimensional Quality and Quality Floors

This paper focused on multi-dimensional pricing and examined the implications of capping a single price in such a multi-dimensional pricing scheme. A similar analysis applies to multi-dimensional quality. For many consumer products and services, quality is measured on multiple dimensions. Consider the cellphone market. Relevant quality dimensions include the functionality of the phone itself (the handset), the scope and duration of the warranty, the reliability of the cellular service (reception, dropped calls, etc'), the accessibility and professionalism of the provider's customer service department, the degree of protection afforded to the customer's personal data, the efficacy and fairness of the contractually-specified dispute resolution mechanism, etc. The level of transparency (or disclosure) about any of these features is yet another quality dimension.

And, as with price, lawmakers often target a single quality dimension for regulation. Rather than capping a certain price dimensions, lawmakers set minimal acceptable levels, or floors, for certain quality dimensions. Sellers' ability to disclaim implied warranties is restricted by law. The Food and Drug Administration (FDA) regulates certain dimensions of pharmaceutical products. The Consumer Product Safety Commission (CPSF) specifies

minimum safety requirements for certain dimensions of certain consumer products. The unconscionability doctrine is used by courts to regulate dispute resolution mechanisms. Consumer protection law imposes minimum disclosure requirements, bans certain contractual terms, mandates cancellation or withdrawal rights (in certain cases), and so on.

Like price, quality is subject to consumer misperception. Consumers might overestimate a certain quality dimension. For example, a cellphone subscriber might overestimate the coverage provided by the carrier's network. Consumers might also misperceive the utility associated with a certain quality dimension. For example, the cellphone subscriber might underestimate the likelihood of traveling to other parts of the country and, therefore, underestimate the utility from broad cellular coverage. Quality misperception corresponds to price misperception. And utility misperception affects price and quality in a similar way. Accordingly, the positive and normative implications of quality floors, as a function of the underlying misperception, can be studied using a framework similar to the one developed in this paper.

References

Agarwal, Sumit, Chomsisengphet, Souphala, Mahoney, Neale and Stroebel, Johannes (2013), Regulating Consumer Financial Products: Evidence from Credit Cards, NBER Working Paper 19484.

Armstrong, Mark and Vickers, John (2012), Consumer Protection and Contingent Charges, Journal of Economic Literature, 50, 477-93.

Bar-Gill, Oren (2006), Bundling and Consumer Misperception, University of Chicago Law Review, 73, 33-61.

Bar-Gill, Oren and Bubb, Ryan (2012), Credit Card Pricing: The Card Act and Beyond, Cornell Law Review, 97, 967-1018.

DellaVigna, Stefano and Malmendier (2004), Ulrike, Contract Design and Self-Control: Theory and Evidence, Quarterly Journal of Economics, 119, 353-402.

Farrell, Joseph (2008), Some Welfare Analytics of Aftermarkets, working paper.

Farrell, Joseph and Klemperer, Paul (2007), Coordination and Lock-In: Competition with Switching Costs and Network Effects, in Armstrong, Mark and Porter, Rob (eds.), Handbook of Industrial Organization, Vol. 3 (2007).

Gabaix, Xavier and Laibson, David (2006), Shrouded Attributes, Consumer Myopia and Information Suppression in Competitive Markets, Quarterly Journal of Economics, 121, 505-540.

Heidhues, Paul, and Koszegi, Botond (2010), Exploiting Naivete about Self-Control in the Credit Market, American Economic Review, 100, 2279-2303.

Heidhues, Paul, Koszegi, Botond and Murooka, Takeshi (2012), Inferior Products and Profitable Deception, Working Paper.

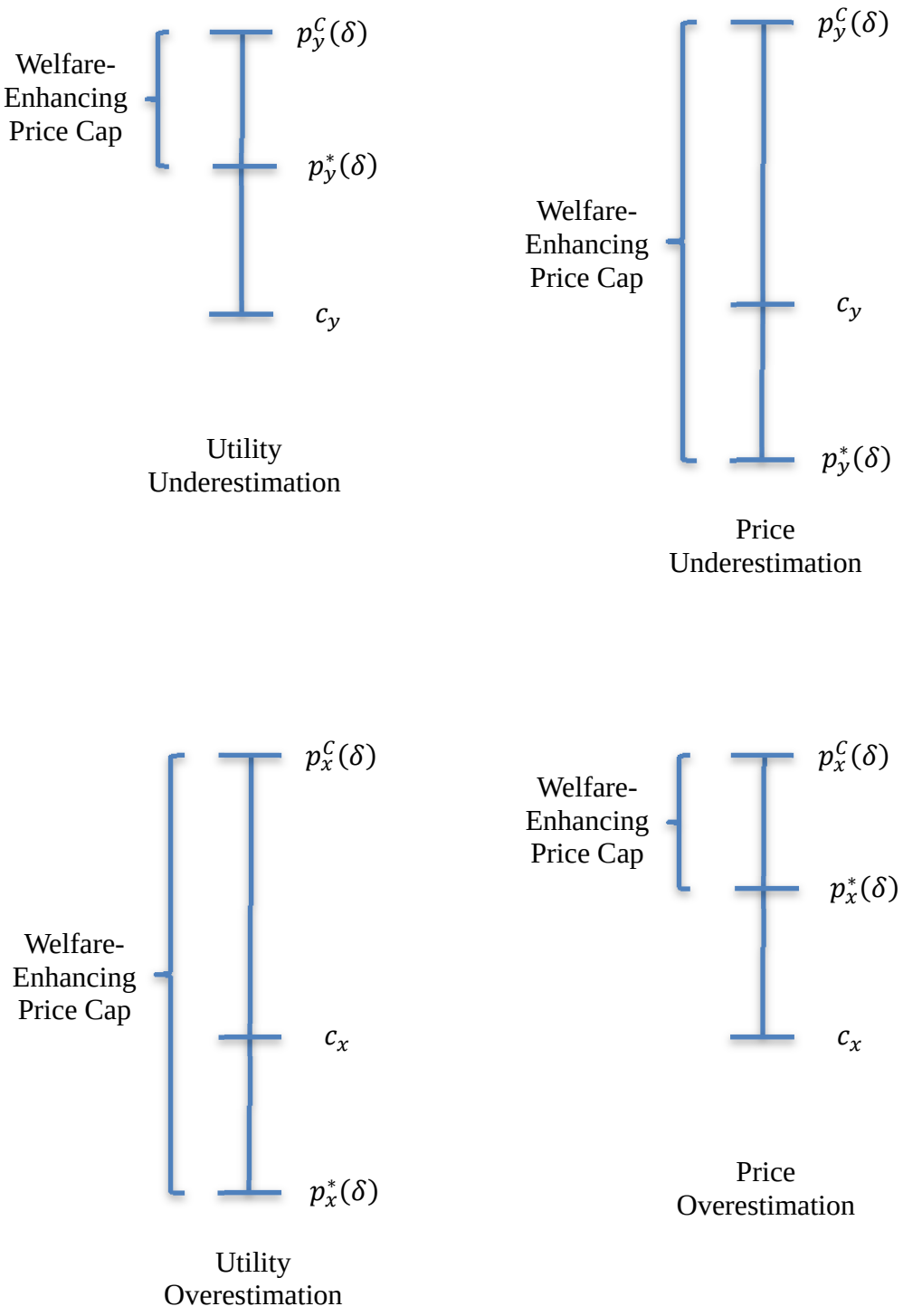

Figure 1: Scope of Welfare-Enhancing Price Caps for Different Types (and Directions) of Misperception