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# Tort Liability and Unawareness\*

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## Abstract

Unawareness is a form of bounded rationality where a person fails to conceive all feasible acts or consequences or to perceive as feasible all conceivable act-consequence links. We study the implications of unawareness for tort law, where relevant examples include the discovery of a new product or technology (new act), of a new disease or injury (new consequence), or that a product can cause an injury (new link). We argue that negligence has an important advantage over strict liability in a world with unawareness—negligence, through the stipulation of due care standards, spreads awareness about the updated probability of harm.

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Keywords: negligence, strict liability, tort law, unawareness.

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# 1 Introduction

A central question in the field of law and economics is whether negligence or strict liability is the more efficient tort liability rule. Under negligence, a victim can recover damages for harm caused by the activity of an injurer who failed to take reasonable care when engaging in the activity. Under strict liability, by contrast, a victim can recover damages for harm caused by the activity of an injurer irrespective of whether the injurer took reasonable care. The relative efficiency of the two rules is customarily measured by the Kaldor-Hicks criterion.

A bedrock result in the economic analysis of tort law is that, in the case of unilateral accidents with fixed activity levels, negligence and strict liability are equally efficient, provided that, in the case of negligence, the court properly sets the due care standard (the legal standard for what constitutes reasonable care) (Shavell, 1987).<sup>1</sup> This equivalence result, however, presents something of a puzzle in light of two facts about negligence. First, negligence is the dominant rule in Anglo-American law.<sup>2</sup> Second, negligence is the more costly rule to administer, because the court must determine the due care standard and adjudicate whether the standard was met. The puzzle is that if negligence and strict liability are equally efficient but negligence is more costly to administer, why is negligence the dominant rule?

The negligence puzzle has led researchers to revisit the equivalence result by exploring departures from the standard accident model, which is based on the expected utility framework and the Bayesian paradigm. For instance, Teitelbaum (2007) and Chakravarty and Kelsey (2017) explore ambiguity (Knightian uncertainty). They assume that the relevant parties are Choquet expected utility maximizers with neo-additive beliefs about accident risk, and they find that this breaks the equivalence in favor of negligence.<sup>3</sup>

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<sup>1</sup>In unilateral accidents, the injurer, but not the victim, can take care to reduce expected harm. In unilateral accidents with fixed activity levels, the injurer affects expected harm only through her level of care (and not through her level of activity). The equivalence result also holds in the case of bilateral accidents with fixed activity levels, provided that strict liability is coupled with the defense of contributory negligence.

<sup>2</sup>In modern Anglo-American law, strict liability applies only in a handful of accident cases, including cases involving abnormally dangerous activities or products with manufacturing defects (Dobbs et al., 2011, § 2). Indeed, certain accident cases that were traditionally governed by strict liability are now governed by negligence, including cases involving products with a design or warning defect (Dobbs et al., 2011, § 450).

<sup>3</sup>The neo-additive Choquet expected utility model was developed by Chateauneuf et al. (2007). Franzoni (2017) models ambiguity aversion according to the smooth model of Klibanoff et al. (2005). He finds that strict liability dominates negligence when the injurer has lower degrees of uncertainty aversion than the

In this paper, we explore *unawareness*. Unawareness is the failure to conceive or perceive the entire state space. It is a form of bounded rationality in which a person fails to conceive all available acts or potential consequences or fails to perceive as feasible all conceivable act-consequence links. Unawareness creates the possibility of *growing awareness*—the expansion of the state space when a person discovers a new act, consequence, or act-consequence link. Examples relevant to tort law include the discovery of a new product or technology (new act), the discovery of a new disease or injury (new consequence), or the discovery that a known product can cause a known injury (new act-consequence link).

We study the implications of unawareness for tort law, and specifically for the negligence versus strict liability debate. To model unawareness and growing awareness, which requires a theory of how beliefs update as the state space expands, we adopt the *reverse Bayesian* approach of Karni and Vierø (2013). Karni and Vierø posit that as a person becomes aware of new acts, consequences, or act-consequence links, her beliefs update in a way that preserves the relative likelihoods of events in the original state space. More specifically, they postulate that (i) in the case of a new act or consequence, probability mass shifts proportionally away from the events in the original state space to the new events in the expanded state space, and (ii) in the case of a new act-consequence link, null events in the original state space become nonnull, and probability mass shifts proportionally away from the original nonnull events to the original null events that become nonnull.<sup>4</sup>

We argue that negligence has an important advantage over strict liability in a world with unawareness and growing awareness. Under either liability rule, a tort litigation involving a new act, consequence, or act-consequence link makes the world aware of a new possibility of harm. Under negligence, however, the litigation provides the world with more information. In particular, the court’s stipulation of a new due care standard serves as a knowledge transmission mechanism, providing the world with information about the updated probability of harm. This information is necessary for either rule to induce the injurers of the world to

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victim and can formulate more precise estimates of the probability of harm, but that negligence dominates strict liability when harm is dispersed on a very large number of victims, irrespective of the parties’ respective degrees of uncertainty aversion.

<sup>4</sup>A null event is an event believed to have zero probability.

take efficient care. Negligence provides this information to injurers. Under strict liability, they would have to expend additional resources to develop this information on their own.

**Related literature.** To our knowledge, this paper is the first to incorporate unawareness into the economic analysis of tort law. As such, we contribute to the tort law and economics literature and to the unawareness literature, both of which are too vast to review here. Surveys of the tort law and economics literature, which was pioneered by Brown (1973),<sup>5</sup> include Shavell (2007), Schäfer and Müller-Langer (2009), and more recently Arlen (2017).

Within the tort law and economics literature, the papers closest to ours include the three described above that explore the implications of ambiguity for tort law (Teitelbaum, 2007; Chakravarty and Kelsey, 2017; Franzoni, 2017).<sup>6</sup> Although ambiguity and unawareness are distinct phenomena, both are types of uncertainty that the standard accident model does not admit.<sup>7</sup> Hence, we share a common enterprise with the papers on tort law and ambiguity—we enrich the standard accident model to allow the parties to face not just risk but rather a more profound and realistic type of uncertainty, and we explore the implications of such uncertainty for the debate over tort liability rules.

We also share connections with Currie and MacLeod (2014) and Feess and Wohlschlegel (2006). Currie and MacLeod (2014) develop an alternative to the standard accident model that makes use of state-space representations, which they dub "Savage Tables," to model the decision problems faced by an injurer (who is fully aware of the state space) under different tort liability rules. They apply their model to argue, *inter alia*, that negligence provides better incentives than strict liability in the case of the Good Samaritan.

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<sup>5</sup>Other early contributions include Diamond (1974a,b), Green (1976), and Shavell (1980).

<sup>6</sup>Also related are the papers that explore the implications of risk aversion for tort law. In the seminal paper on the topic, Shavell (1982) shows that strict liability is superior when the injurer is risk neutral and the victim is risk averse, while negligence is superior in the opposite case. Franzoni (2017, n. 10) reviews other papers on optimal tort liability rules under risk aversion and related contributions.

<sup>7</sup>"Under ambiguity, the agent still conceives of the space of all relevant contingencies but has difficulties to evaluate them probabilistically. Under unawareness, the agent cannot even conceive all relevant contingencies" (Schipper, 2014, p. 1). Researchers currently are exploring connections between ambiguity and unawareness. For instance, Dominiak and Tserenjigmid (2018) generalize the model of Karni and Vierø (2013) such that the decision maker perceives ambiguity in the wake of growing awareness. We draw a similar connection between growing awareness and ambiguity in Section 4.4, where we discuss circumstances under which reverse Bayesian updating results in ambiguity about the updated probability of harm.

Feess and Wohlschlegel (2006) compare negligence and strict liability in world with asymmetric information. They model a situation where some injurers have better information than others and the court about a variable  $\theta$  that characterizes the efficient level of care, and where the court does not know which injurers are informed or uninformed. They show that under certain conditions the court can learn  $\theta$  (and thus the efficient level of care) over time by imperfectly observing the injurer's level of care in a large number of cases, and that under negligence the uninformed injurers can in turn learn  $\theta$  over time by observing the evolution of the court's due care standard. They further note that under strict liability the court lacks an instrument to transmit the information about  $\theta$  to the uninformed injurers. We show that negligence has a similar advantage over strict liability in a world with unawareness.

Surveys of the unawareness literature, which was pioneered by Fagin and Halpern (1988),<sup>8</sup> include Schipper (2014) (which offers a "gentle introduction") and Schipper (2015) (which provides an extended review). Karni and Vierø (2013) are among the pioneers of the choice-theoretic approach (i.e., the state-space approach) to modeling unawareness.<sup>9</sup> A handful of papers apply unawareness models to study legal topics. The bulk of these focus on contracts. For example, Zhao (2011) argues that unawareness may explain the existence of force majeure clauses in contracts; Filiz-Ozbay (2012) posits asymmetric awareness as a reason for the incompleteness of contracts; and Auster (2013) introduces asymmetric unawareness into the canonical moral hazard model and analyzes the properties of the optimal contract.<sup>10</sup>

We also contribute to the relatively nascent but rapidly growing behavioral law and economics literature. Sunstein (1997), Jolls et al. (1998), and Korobkin and Ulen (2000) were

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<sup>8</sup>Other early contributions include Modica and Rustichini (1994, 1999), Dekel et al. (1998), Halpern (2001), Heifetz et al. (2006), and Halpern and Rêgo (2008).

<sup>9</sup>A number of subsequent papers build on their approach. For instance, Grant et al. (2017) invoke their approach to model learning by experimentation in a world with unawareness; Karni and Vierø (2017) extend their model to the case where the decision maker anticipates her growing awareness; and Dominiak and Tserenjigmid (2018) generalize their model such that the decision maker perceives ambiguity in the wake of growing awareness. Karni and Vierø (2013, 2017) and Dominiak and Tserenjigmid (2018) survey the papers in the literature that take a choice-theoretic approach to modeling unawareness.

<sup>10</sup>In addition, Board and Chung (2011) argue that asymmetric unawareness provides a justification for the *contra proferentem* doctrine of contract interpretation, which provides that ambiguous terms in a contract should be construed against the drafter; Grant et al. (2012) study aspects of differential awareness that give rise to contractual disputes; and von Thadden and Zhao (2012, 2014) study the properties of optimal contracts under moral hazard when the agent may be partially unaware of her action space.

early calls for the modification of standard law and economics models to reflect advances in behavioral economics and decision theory.<sup>11</sup> Halbersberg and Guttel (2014) and Luppi and Parisi (2018) provide surveys of behavioral models of tort law.

The remainder of the paper is organized as follows. Section 2 presents the accident model—a unilateral accident model featuring multiple activities with fixed levels—and derives the equivalence result. Section 3 presents the unawareness model and provides relevant examples of new acts, new consequences, and new act-consequence links. Section 4 compares and contrasts negligence and strict liability in a world with unawareness. It considers a simplified world with two acts, two consequences, quadratic care costs, and linear expected harm reduction, and separately analyzes the cases of a new act, a new consequence, and a new act-consequence link. Section 5 extends the analysis to a more general world with  $m$  acts,  $n$  consequences, convex care costs, and convex expected harm reduction. Section 6 discusses the results and suggests directions for future research. The Appendix collects the proofs of the propositions and corollaries stated but not proved in the body of the paper.

## 2 The Accident Model

There are two representative agents: an injurer and a victim. Both are risk neutral subjective expected utility maximizers. The agents are strangers and not parties to a contract or market transaction, and transaction costs are sufficiently high to preclude Coasian bargaining.

The injurer has available  $m \geq 2$  activities,  $f_1, \dots, f_m$ . Each activity has the potential to cause harm to the victim, though we assume the outcomes are independent across activities. That is, we assume the activities are independent experiments, akin to  $m$  one-armed bandits. We refer to this assumption below as *act independence*.<sup>12</sup>

There are  $n \geq 2$  potential degrees of harm,  $z_1, \dots, z_n$ , where  $z_j \geq 0$  for all  $j = 1, \dots, n$ . Activity  $f_i$  causes harm  $z_j$  with probability  $\pi_{ij}$ , where  $\sum_{j=1}^n \pi_{ij} = 1$  for all  $i = 1, \dots, m$ .

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<sup>11</sup>Sunstein (2000) and Parisi and Smith (2005) are edited volumes that collect early papers in the literature. Zamir and Teichman (2014) and Teitelbaum and Zeiler (2018) are more recent volumes.

<sup>12</sup>While act independence is a reasonable assumption in many settings, there undoubtedly are settings in which it is not. We explore the implications of relaxing the act independence assumption in Section 4.4.

Thus, activity  $f_i$ 's expected harm is  $\sum_{j=1}^n \pi_{ij} z_j$ . In the absence of unawareness, the agents have correct beliefs about each harm probability  $\pi_{ij}$ .

The injurer engages in each available activity. For each activity  $f_i$ , the injurer, but not the victim, can take care to reduce the activity's expected harm to the victim. The injurer chooses a level of care  $x_i \geq 0$  having cost  $c(x_i)$ . Being careless is costless,  $c(0) = 0$ , and the marginal cost of care is positive and increasing:  $c'(x_i) > 0$  and  $c''(x_i) > 0$  for all  $x_i$ . Taking care reduces the activity's expected harm at a nonincreasing rate:  $h_i(x_i) \equiv \sum_{j=1}^n \pi_{ij} z_j \tau(x_i)$ , where  $\tau(x_i) \in (0, 1]$  for all  $x_i$  with  $\tau(0) = 1$  and where  $\tau'(x_i) < 0$  and  $\tau''(x_i) \geq 0$  for all  $x_i$ .<sup>13</sup>

If activity  $f_i$  causes harm, the victim may be entitled to damages from the injurer, depending on the applicable tort liability rule. Under negligence, the victim is entitled to damages equal to the harm if the injurer's level of care was below the due care standard for the activity,  $\bar{x}_i$ , which is stipulated by the court.<sup>14</sup> Under strict liability, the victim is entitled to damages equal to the harm irrespective of the injurer's level of care. We assume the injurer has the ability to pay any and all damages to which the victim may be entitled.

The social goal is to minimize the total social costs of the injurer's activities (the sum of the costs of care and the expected harms):

$$\underset{x_1, \dots, x_m \geq 0}{\text{minimize}} \quad \sum_{i=1}^m c(x_i) + h_i(x_i).$$

The solution  $\tilde{x} = (\tilde{x}_1, \dots, \tilde{x}_m)$  is given implicitly by the first order conditions

$$c'(\tilde{x}_i) = -h'_i(\tilde{x}_i), \quad i = 1, \dots, m,$$

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<sup>13</sup>We assume  $c(\cdot)$  and  $\tau(\cdot)$  are common knowledge but not activity specific. The latter assumption is without loss of generality given the former assumption; we make the latter assumption to simplify the notation.

<sup>14</sup>Following in the tradition of the tort law and economics literature, we model the due care standard as a precise stipulation. In reality, the due care standard may be less specific. For a discussion on the specificity of the due care standard at common law, see Dobbs et al. (2011, § 145).

and is given explicitly by

$$\tilde{x}_i = \xi^{-1} \left( \sum_{j=1}^n \pi_{ij} z_j \right), \quad i = 1, \dots, m,$$

where  $\xi^{-1}$  denotes the inverse of  $\xi(x_i) \equiv -c'(x_i)/\tau'(x_i)$ .<sup>15</sup> We refer to  $\tilde{x}_i$  as the efficient level of care for activity  $f_i$ . It is the level of care at which the marginal cost of care equals the marginal benefit (the marginal reduction in expected harm).

Under strict liability, the injurer's problem is identical to the social goal. This is because strict liability forces the injurer to internalize the total social costs of her activities. Hence, strict liability induces the injurer to take efficient care in each activity.

Under negligence, the injurer's problem is

$$\begin{aligned} & \text{minimize}_{x_1, \dots, x_m \geq 0} && \sum_{i=1}^m c(x_i) + h_i(x_i) \chi(x_i < \bar{x}_i), \\ & \text{where} && \chi(x_i < \bar{x}_i) \equiv \begin{cases} 1 & \text{if } x_i < \bar{x}_i \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

and where  $\bar{x}_i$  is the due care standard for activity  $f_i$ . If the court sets  $\bar{x}_i = \tilde{x}_i$  for all  $i$ , then the injurer takes efficient care in each activity. The reason is twofold. First, the injurer will not take more than the efficient level of care, because she faces no liability if her level of care equals or exceeds the efficient level. Second, the injurer will not take less than the efficient level of care, because then she faces strictly liability, which induces her to take efficient care.

The equivalence result follows immediately from the foregoing.

**Theorem 1 (Equivalence Result)** *The injurer will take efficient care in each activity under either negligence or strict liability, provided that, in the case of negligence, the court sets the due care standard for each activity equal to the efficient level of care for that activity.*

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<sup>15</sup>Note that  $\xi'(x_i) = \frac{c'(x_i)t''(x_i) - t'(x_i)c''(x_i)}{[t'(x_i)]^2} > 0$  for all  $x_i$ ; hence  $\xi$  is invertible.

### 3 The Unawareness Model

We model unawareness and growing awareness à la Karni and Vierø (2013). The primitives of the model are a finite, nonempty set  $F$  of feasible *acts* and a finite, nonempty set  $Z$  of feasible *consequences*. In our setting, the feasible acts are the injurer's available activities and the feasible consequences are the potential harms to the victim.

States are functions from the set of acts to the set of consequences. A state assigns a consequence to each act. The set of all possible states,  $Z^F$ , defines the *conceivable state space*. With  $m$  acts and  $n$  consequences, there are  $n^m$  conceivable states.

The agents and the court (collectively, the *parties*) originally conceive the sets of acts and consequences to be  $F = \{f_1, \dots, f_m\}$  and  $Z = \{z_1, \dots, z_n\}$ . The conceivable state space is  $Z^F = \{s_1, \dots, s_{n^m}\}$ , where each state  $s \in Z^F$  is a vector of length  $m$ , the  $i$ th element of which,  $s^i$ , is the consequence  $z_j \in Z$  produced by act  $f_i \in F$  in that state of the world.

An act-consequence link, or *link*, is a causal relationship between an act and a consequence. The conceivable state space admits all conceivable links. However, the parties may perceive one or more links as infeasible, which brings them to nullify the states that admit such link. We refer to these as null states. Taking only the nonnull states defines the *feasible state space*,  $S \equiv Z^F \setminus N$ , where  $N \subset Z^F$  is the set of null states. When  $N \neq \emptyset$ , there are  $\prod_{i=1}^m (n - \nu_i)$  feasible states, where  $\nu_i$  denotes the number of nullified links involving act  $f_i$ .

The parties have common beliefs represented by a probability measure  $p$  on the conceivable state space,  $Z^F$ . The support set of  $p$  is the feasible state space,  $S$ . That is, the parties assign nonzero probability to each nonnull state and zero probability to each null state.

The parties may initially fail to conceive one or more acts or consequences or to perceive as feasible one or more conceivable links. We refer to such failures of conception or perception as *unawareness*. However, the parties may later discover a new act or consequence, which expands both the feasible state space and the conceivable state space, or a new link, which expands the feasible state space but not the conceivable state space. We refer to such discoveries and expansions as *growing awareness*.<sup>16</sup>

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<sup>16</sup>To be clear, by "new" we mean "not previously conceived" in the case of acts and consequences, and

To illustrate, suppose  $S = Z^F$  and the parties discover a new consequence,  $z_{n+1}$ . Then the set of potential harms becomes  $\widehat{Z} = Z \cup \{z_{n+1}\}$  and the feasible and conceivable state spaces both expand to  $\widehat{S} = \widehat{Z}^F = \{s_1, \dots, s_{(n+1)^m}\}$ , where each state remains a vector of length  $m$ . Alternatively, suppose the parties discover a new act,  $f_{m+1}$ . Then the set of available activities becomes  $\widehat{F} = F \cup \{f_{m+1}\}$  and the feasible and conceivable state spaces both expand to  $\widehat{S} = \widehat{Z}^{\widehat{F}} = \{s_1, \dots, s_{n(m+1)}\}$ , where each state now is a vector of length  $m+1$ . Lastly, suppose  $S \subset Z^F$  because the parties initially perceive as infeasible the link from  $f_1$  to  $z_n$ . Discovery of the link from  $f_1$  to  $z_n$  does not alter the conceivable state space, but the feasible state space expands to coincide with the conceivable state space:  $\widehat{S} = Z^F$ .<sup>17</sup>

In the wake of growing awareness, the parties' beliefs update in a way that preserves the relative likelihoods of the events in the original feasible state space (which are the nonnull events in the original conceivable state space). In each case of growing awareness, probability mass shifts proportionally away from the events in the original feasible state space to the new events in the expanded feasible state space. In the case of a new act or consequence, the new events in the expanded feasible state space are also new events in the expanded conceivable state space. In the case of a new link, the new events in the expanded feasible state space are the null events in the original conceivable state space that become nonnull.

Karni and Vierø refer to this updating as *reverse Bayesianism*. Let  $\widehat{p}$  denote the parties' updated beliefs. Formally, reverse Bayesianism implies: (i) in the case of a new consequence or link,  $p(s)/p(t) = \widehat{p}(s)/\widehat{p}(t)$  for all  $s, t \in S$ ; and (ii) in the case of a new act,  $p(s)/p(t) = \widehat{p}(E(s))/\widehat{p}(E(t))$  for all  $s, t \in S$ , where  $E(s)$  denotes the event in  $\widehat{S}$  that corresponds to state  $s$  in  $S$ ; that is,  $E(s) \equiv \{t \in \widehat{S} : t^i = s^i \text{ for all } i \neq m+1\}$  (assuming the new act is  $f_{m+1}$ ).

The act independence assumption implies additional restrictions on  $\widehat{p}$ . Let  $A_i(z_j) \subset \widehat{S}$  denote the event that  $f_i$  yields  $z_j$ ; that is,  $A_i(z_j) \equiv \{t \in \widehat{S} : t^i = z_j\}$ . We refer to events of this type as *act events*. Act independence implies  $A_i(z_j) \perp A_{i'}(z_{j'})$  for all  $i$  and  $i'$  where  $i \neq i'$  and all  $j$  and  $j'$ .<sup>18</sup> Take any event  $E \subseteq \widehat{S}$ . We can express each state  $s = (s^1, \dots, s^m) \in E$

<sup>17</sup>"previously conceived but perceived as infeasible" in the case of links.

<sup>17</sup>Section 4 contains illustrative depictions of conceivable and feasible state spaces and their expansion due to the discovery of new acts, consequences, and links.

<sup>18</sup>The symbol  $\perp$  denotes statistical independence.

as the intersection of a unique collection of independent act events:  $s = \bigcap_i A_i(s^i)$ . It follows that  $\widehat{p}(s) = \prod_i \widehat{p}(A_i(s^i))$  for all  $s \in E$ . Observe that growing awareness, whether it entails a new act, consequence, or link, gives rise to a new event  $\Delta = \widehat{S} \setminus S$ . Thus, in each case of growing awareness, act independence implies  $\widehat{p}(s) = \prod_i \widehat{p}(A_i(s^i))$  for all  $s \in \Delta$ .

### 3.1 Examples of Growing Awareness

Instances of growing awareness that are relevant to tort law include the discovery of a new and potentially harmful product or technology (new act), the discovery of a new disease or injury (new consequence), or the discovery that a known product can cause a known injury in a previously unknown way (new link). We list below a few illustrative examples.

#### 3.1.1 New Acts

**Fracking.** Modern day hydraulic fracturing, or "fracking," was developed by George Mitchell in the late 1990s, though its origins can be traced to the 1860s (Gold, 2014). In *Ely v. Cabot Oil and Gas Corporation*, a Pennsylvania jury found in favor of nine plaintiffs on claims that the defendant's fracking activity at two natural gas wells in Susquehanna County in the mid-2000s was negligent and caused the plaintiffs' compensable nuisance injuries by interfering with and damaging the plaintiffs' access to water and their enjoyment of their property. The gravamen of the plaintiffs' complaint was that the defendant's fracking activity negligently permitted methane to flow into underground aquifers that wound up polluting the plaintiffs' water wells.<sup>19</sup> Less than two years after the jury verdict in 2016, and less than three months after the case was finally settled in 2017, the defendant and 25 other major oil and gas companies, under the auspices of the American Petroleum Institute, announced that they were launching a new program, called the Environmental Partnership, focused on reducing emissions of methane and other pollutants from the natural gas sector (Henry, 2017).

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<sup>19</sup>See 2017 WL 1196510 (M.D. Pa. 2017) (denying defendant's motion for judgment as a matter of law but granting its motion for a new trial). The case settled before the second trial.

**Cyberbullying.** The World Wide Web was invented by Tim Berners-Lee in 1989 (Gillies and Cailliau, 2000). In *D.C. v. R.R.*, a Los Angeles high school student brought an action against several of his fellow students—who had posted messages at his Web site making derogatory comments about his perceived sexual orientation and threatening him with bodily harm—claiming, *inter alia*, defamation and intentional infliction of emotional distress.<sup>20</sup> According to the Cyberbullying Research Center (2016), the rate of cyberbullying offending among U.S. middle and high school students decreased by 40 percent in the six years after the key decision in the case by the California Court of Appeal in 2010.

### 3.1.2 New Consequences

**HIV/AIDS.** The first HIV/AIDS cases were reported in the United States in 1981 (U.S. Centers for Disease Control and Prevention, 2011). In *Quintana v. United Blood Services*, a Denver jury held the defendant liable to the plaintiff for negligently supplying her with HIV contaminated blood in 1983 (Talavera, 1993).<sup>21</sup> In the three decades since the case was filed, U.S. blood banks, through "the use of donor educational material, specific deferral questions, and advances in HIV donor testing . . . have reduced the risk of HIV transmission from blood transfusion from about 1 in 2500 units . . . to a current estimated residual risk of about 1 in 1.47 million transfusions" (U.S. Food and Drug Administration, 2015, p. 2).

**Mad cow disease.** Mad cow disease, also known as bovine spongiform encephalopathy (BSE), was first discovered in the United Kingdom in 1986 (Collee and Bradley, 1997). In 2008, Ridley Inc., a Winnipeg cattle feed supplier, settled a class action by Canadian cattle farmers claiming that the defendant negligently supplied them with BSE contaminated feed in the early 1990s (Dowd, 2008). Still pending is a related class action against the Canadian government for negligently allowing BSE infected cattle to be imported into Canada in the late 1980s and used for feed ingredients in the early 1990s (Kienlen, 2017). In 2007, two

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<sup>20</sup>See 182 Cal. App. 4th 1190 (2010) (affirming the trial court's denial of defendants' motion to strike under California's strategic lawsuit against public participation (SLAPP) statute).

<sup>21</sup>The jury also held the defendant liable to the plaintiff's husband for negligent infliction of emotional distress and loss of consortium. The case was initially filed in the late 1980s and the final verdict was rendered in 1992 (Talavera, 1993).

years after the class action was filed, the Canadian government implemented an enhanced feed ban aimed at preventing the spread of BSE (Stephenson, 2015).

### 3.1.3 New Links

**Agent Orange and cancer.** Agent Orange, a chemical herbicide, was developed in the 1940s and used by the U.S. military as part of its herbicidal warfare program, Operation Ranch Hand, during the Vietnam War (Schuck, 1987). The first recorded case of cancer hails from ancient Egypt, and the origin of the word cancer is credited to the ancient Greek physician Hippocrates (Sudhakar, 2009). In *In re Agent Orange Product Liability Litigation*, Vietnam veterans brought a class action against the manufacturers of Agent Orange alleging, *inter alia*, that their exposure to Agent Orange in Vietnam resulted in a variety of cancers and other diseases in the veterans and birth defects in their children.<sup>22</sup> The class action was filed in 1979. In 1983, seven months before the parties reached a settlement, Dow Chemical Company, the lead defendant in the class action, announced that it was permanently discontinuing production of 2,4,5-trichlorophenoxyacetic acid, the component of Agent Orange that was responsible for its toxicity (Holusha, 1983; American Chemical Society, 1985).

**American football and CTE.** The National Football League (NFL) was founded in 1920 (Crepeau, 2014). Chronic traumatic encephalopathy (CTE), a neurodegenerative brain disease found in people with a history of repetitive head injuries, was first reported in boxers 1928 and in NFL players in 2005 (Lindsley, 2017). In *In re National Football League Players Concussion Injury Litigation*, retired NFL players brought a class action against the NFL alleging that the league had failed to take reasonable actions to protect the players from CTE and other chronic risks of head injuries in football.<sup>23</sup> The class action was filed in 2011. Seven months before the parties reached a settlement in 2013, the NFL announced a new concussion protocol, which includes guidelines for sideline evaluation and rules on preseason education, baseline testing, and the establishment of personnel to conduct evaluations (Flynn, 2016).

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<sup>22</sup>See 597 F. Supp. 740 (E.D.N.Y. 1984). The case was settled for \$180 million.

<sup>23</sup>See 821 F.3d 410 (3d Cir. 2016). The case was settled for approximately \$1 billion.

## 4 Illustrative Results

In this section and the next, we compare and contrast negligence and strict liability in a world with unawareness. In both sections, we assume the parties are fully rational apart from unawareness. We further assume that when the parties are unaware of an act, consequence, or link, their beliefs, although incorrect with respect to the absolute likelihoods of events, are nevertheless correct with respect to the relative likelihoods of nonnull events. Without this assumption, the parties could not have correct beliefs when they become fully aware, which would be inconsistent with the standard accident model.

In this section, we consider a simplified world with two acts,  $F = \{f_1, f_2\}$ ; two consequences,  $Z = \{z_1, z_2\}$ , where  $z_1 = 0$  and  $z_2 > 0$ ; quadratic care costs,  $c(x_i) = (x_i)^2$ ; and linear expected harm reduction,  $\tau(x_i) = (1 - x_i)$ .<sup>24</sup> Our analysis of this simplified world illustrates all of the main ideas of the paper. In the next section, we show that the results extend to a more general world with  $m$  acts,  $n$  consequences, convex care costs, and convex expected harm reduction.<sup>25</sup>

With two acts (activities),  $f_1$  and  $f_2$ , and two consequences (harms),  $z_1 = 0$  and  $z_2 > 0$ , the conceivable state space,  $Z^F$ , comprises four states:  $s_1 = (0, 0)$ ,  $s_2 = (0, z_2)$ ,  $s_3 = (z_2, 0)$ , and  $s_4 = (z_2, z_2)$ . Let  $p_k \equiv p(s_k)$ ,  $k = 1, \dots, 4$ , denote the parties' common beliefs on  $Z^F$ . We can depict the original conceivable state space and the parties' beliefs as follows:

$p$	$p_1$	$p_2$	$p_3$	$p_4$
$F \setminus Z^F$	$s_1$	$s_2$	$s_3$	$s_4$
$f_1$	0	0	$z_2$	$z_2$
$f_2$	0	$z_2$	0	$z_2$

<sup>24</sup>To preserve the condition  $\tau(x_i) > 0$  for all  $x_i$ , we assume  $x_i \in [0, 1)$  in this section.

<sup>25</sup>In this section and the next, we assume  $c(\cdot)$  and  $\tau(\cdot)$  are the same for all activities. Of course, provided the world knows them all, we can relax this assumption and assume heterogeneity across activities.

## 4.1 New Link

We start with the case of a new link. We assume the parties initially perceive activity  $f_1$  as safe and activity  $f_2$  as risky. That is, we assume they initially perceive the event  $\{s_3, s_4\}$  as infeasible (null). This implies  $p_3 = p_4 = 0$ . We can depict the original feasible state space,  $S \subset Z^F$ , as follows:

$p$	$p_1$	$p_2$
$F \setminus S$	$s_1$	$s_2$
$f_1$	0	0
$f_2$	0	$z_2$

Given  $S$  and  $p$ , the efficient levels of care are

$$\tilde{x}_1 = 0 \quad \text{and} \quad \tilde{x}_2 = \frac{p_2 z_2}{2}.$$

Under negligence, the court stipulates  $\bar{x}_1 = \tilde{x}_1$  and  $\bar{x}_2 = \tilde{x}_2$  as the due care standards for  $f_1$  and  $f_2$ , respectively.

Suppose the parties discover that activity  $f_1$  is risky. In particular, suppose that the injurer engages in  $f_1$ , that it results in harm  $z_2$ , and that the victim brings a tort suit against the injurer before the court. The feasible state space expands to coincide with the conceivable state space,  $\hat{S} = Z^F$ , and the parties update their beliefs from  $p$  to  $\hat{p}$ :

$\hat{p}$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$
$F \setminus \hat{S}$	$s_1$	$s_2$	$s_3$	$s_4$
$f_1$	0	0	$z_2$	$z_2$
$f_2$	0	$z_2$	0	$z_2$

We assume that, by virtue of the suit, the parties learn that activity  $f_1$  yields harm  $z_2$  with probability  $\delta > 0$ . The fact is the parties have the incentive to expend resources to

develop this knowledge. As the Hand formula makes plain,<sup>26</sup> the probability of harm is an essential component of a negligence case. Even in a strict liability case, the probability of harm is relevant to the issues of foreseeability and proximate cause.<sup>27</sup>

Note that  $\delta$  is the total probability of the new states in the expanded feasible state space. It is a measure of the likelihood of the event of which the parties were previously unaware. Thus, we interpret  $\delta$  as the *degree of unawareness*.

By reverse Bayesianism,

$$\frac{p_1}{p_2} = \frac{\hat{p}_1}{\hat{p}_2}.$$

In addition,  $\delta = \hat{p}_3 + \hat{p}_4$  (by definition) and  $\hat{p}_1 + \hat{p}_2 + \hat{p}_3 + \hat{p}_4 = 1$  (by the unit measure axiom on  $\hat{S}$ ). Moreover, by act independence,

$$\hat{p}_3 = (\hat{p}_3 + \hat{p}_4)(\hat{p}_1 + \hat{p}_3) \quad \text{and} \quad \hat{p}_4 = (\hat{p}_3 + \hat{p}_4)(\hat{p}_2 + \hat{p}_4).$$

It follows that the updated probability measure  $\hat{p}$  is given by:

**Proposition 1**  $\hat{p}_1 = (1 - \delta)p_1$ ,  $\hat{p}_2 = (1 - \delta)p_2$ ,  $\hat{p}_3 = \delta p_1$ , and  $\hat{p}_4 = \delta p_2$ .

Note that  $p$  is the Bayesian update of  $\hat{p}$  conditional on the event  $\{s_1, s_2\}$ ; hence the term reverse Bayesianism.

Given  $\hat{S}$  and  $\hat{p}$ , the efficient levels of care are

$$\hat{x}_1 = \frac{(\hat{p}_3 + \hat{p}_4)z_2}{2} = \frac{\delta z_2}{2} \quad \text{and} \quad \hat{x}_2 = \frac{(\hat{p}_2 + \hat{p}_4)z_2}{2} = \frac{p_2 z_2}{2}.$$

Note that  $\hat{x}_1 > \tilde{x}_1$  but  $\hat{x}_2 = \tilde{x}_2$ . Thus, the discovery that  $f_1$  is risky necessitates the stipulation of a new due care standard for  $f_1$  but not for  $f_2$ .

Under negligence, the court stipulates  $\hat{x}_1 = \tilde{x}_1$  as the new due care standard for  $f_1$  and holds the injurer liable to pay damages of  $z_2$  to the victim.<sup>28</sup> This makes the injurers and

<sup>26</sup>See *United States v. Carroll Towing Co.*, 159 F.2d 169 (2d Cir. 1947); Posner (1998).

<sup>27</sup>Alternatively, we could assume the court learns  $\delta$  by virtue of a sequence of suits (cf. Feess and Wohlschlegel, 2006).

<sup>28</sup>Recall that  $\bar{x}_1 = \tilde{x}_1 = 0$  before the parties discover that  $f_1$  is risky. Under negligence, therefore, the

victims of the world aware that  $f_1$  is risky. Moreover, they can deduce  $\delta$  from  $\widehat{x}_1$ . Specifically, they can deduce that  $\delta = 2\widehat{x}_1/z_2$ . As a result, they can learn  $\widehat{p}$  and  $\widehat{h}_1(x_1) = \delta z_2 \tau(x_1)$ , without expending additional resources to learn  $\delta$ . Knowledge of  $\widehat{h}_1(x_1)$  is necessary to induce injurers to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of  $z_2$  to the victim. This makes the injurers and victims of the world aware that  $f_1$  is risky. However, they cannot deduce  $\delta$  or learn  $\widehat{p}$  or  $\widehat{h}_1(x_1)$ . Without knowledge of  $\widehat{h}_1(x_1)$ , strict liability cannot induce injurers to take efficient care.<sup>29</sup>

## 4.2 New Act

We next consider the case of a new act. We assume the original feasible state space coincides with the original conceivable state space (i.e.,  $S = Z^F$ ):

$p$	$p_1$	$p_2$	$p_3$	$p_4$
$F \setminus S$	$s_1$	$s_2$	$s_3$	$s_4$
$f_1$	0	0	$z_2$	$z_2$
$f_2$	0	$z_2$	0	$z_2$

Given  $S$  and  $p$ , the efficient levels of care are

$$\widetilde{x}_1 = \frac{(p_3 + p_4)z_2}{2} \quad \text{and} \quad \widetilde{x}_2 = \frac{(p_2 + p_4)z_2}{2}.$$

Under negligence, the court stipulates  $\bar{x}_1 = \widetilde{x}_1$  and  $\bar{x}_2 = \widetilde{x}_2$  as the due care standards for  $f_1$  and  $f_2$ , respectively.

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injurer will have exercised no care in conjunction with  $f_1$ . However, even if the court does not hold the injurer liable and award damages (perhaps by recognizing a civil ex post facto doctrine which prohibits retroactive application of a due care standard in a negligence suit), our results below would not change, because the world already knows the set of potential harms.

<sup>29</sup>Note that even if injurers have different care cost and expected harm reduction functions—in which case the efficient level of care is different for each injurer—under negligence they still can deduce  $\delta$  from  $\widehat{x}_1$  (as long as  $c(\cdot)$  and  $\tau(\cdot)$  are common knowledge), which enables them to calculate their individual efficient levels of care, without expending additional resources. Under strict liability, by contrast, each injurer would have to expend additional resources (to learn  $\delta$ ) in order to calculate its individual efficient level of care.

Suppose the parties discover a new activity,  $f_3$ , which they perceive as risky. In particular, suppose that the injurer discovers and engages in  $f_3$ , that it results in harm  $z_2$ , and that the victim brings a tort suit against the injurer before the court. The feasible state space expands from four to eight states:

$\widehat{p}$	$\widehat{p}_1$	$\widehat{p}_2$	$\widehat{p}_3$	$\widehat{p}_4$	$\widehat{p}_5$	$\widehat{p}_6$	$\widehat{p}_7$	$\widehat{p}_8$
$\widehat{F} \setminus \widehat{S}$	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$	$s_7$	$s_8$
$f_1$	0	0	$z_2$	$z_2$	0	0	$z_2$	$z_2$
$f_2$	0	$z_2$	0	$z_2$	0	$z_2$	0	$z_2$
$f_3$	0	0	0	0	$z_2$	$z_2$	$z_2$	$z_2$

The expanded feasible state space contains two copies of the original feasible state space, one in which  $f_3$  results in no harm and one in which  $f_3$  results in harm  $z_2$ . Stated differently, the expanded space splits each of the original states into two depending on whether  $f_3$  yields no harm or harm  $z_2$ . For each state in the original feasible state space there is a corresponding event in the expanded feasible state space. In particular, the event  $\{s_1, s_5\} \in \widehat{S}$  corresponds to state  $s_1 \in S$ , the event  $\{s_2, s_6\} \in \widehat{S}$  corresponds to state  $s_2 \in S$ , the event  $\{s_3, s_7\} \in \widehat{S}$  corresponds to state  $s_3 \in S$ , and the event  $\{s_4, s_8\} \in \widehat{S}$  corresponds to state  $s_4 \in S$ .<sup>30</sup>

As before, we assume that, by virtue of the suit, the parties learn that activity  $f_3$  yields harm  $z_2$  with probability  $\delta > 0$ . By reverse Bayesianism,

$$\begin{aligned} \frac{p_1}{p_2} &= \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_2 + \widehat{p}_6}, & \frac{p_1}{p_3} &= \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_3 + \widehat{p}_7}, & \frac{p_1}{p_4} &= \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_4 + \widehat{p}_8}, \\ \frac{p_2}{p_3} &= \frac{\widehat{p}_2 + \widehat{p}_6}{\widehat{p}_3 + \widehat{p}_7}, & \frac{p_2}{p_4} &= \frac{\widehat{p}_2 + \widehat{p}_6}{\widehat{p}_4 + \widehat{p}_8}, & \text{and} & \frac{p_3}{p_4} &= \frac{\widehat{p}_3 + \widehat{p}_7}{\widehat{p}_4 + \widehat{p}_8}. \end{aligned}$$

In addition,  $\delta = \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8$  (by definition) and  $\widehat{p}_1 + \dots + \widehat{p}_8 = 1$  (by the unit measure

<sup>30</sup>Note that the conceivable state space also expands from four to eight states, so  $\widehat{S} = Z^{\widehat{F}}$ .

axiom on  $\widehat{S}$ ). Moreover, by act independence,

$$\begin{aligned}\widehat{p}_5 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8), \\ \widehat{p}_6 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8), \\ \widehat{p}_7 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8), \\ \text{and } \widehat{p}_8 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8).\end{aligned}$$

It follows that the updated probability measure  $\widehat{p}$  is given by:

**Proposition 2**  $\widehat{p}_1 = (1 - \delta)p_1, \quad \widehat{p}_2 = (1 - \delta)p_2, \quad \widehat{p}_3 = (1 - \delta)p_3, \quad \widehat{p}_4 = (1 - \delta)p_4,$   
 $\widehat{p}_5 = \delta p_1, \quad \widehat{p}_6 = \delta p_2, \quad \widehat{p}_7 = \delta p_3, \quad \text{and } \widehat{p}_8 = \delta p_4.$

Given  $\widehat{S}$  and  $\widehat{p}$ , the efficient levels of care are

$$\begin{aligned}\widehat{x}_1 &= \frac{(\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8) z_2}{2} = \frac{(p_3 + p_4) z_2}{2}, \\ \widehat{x}_2 &= \frac{(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8) z_2}{2} = \frac{(p_2 + p_4) z_2}{2}, \\ \text{and } \widehat{x}_3 &= \frac{(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) z_2}{2} = \frac{\delta z_2}{2}.\end{aligned}$$

Thus, the discovery of  $f_3$  necessitates the stipulation of a new due care standard,  $\widehat{x}_3$ , but it does not necessitate the stipulation of a new due care standard for  $f_1$  or  $f_2$ .

Under negligence, the court stipulates  $\widehat{x}_3 = \widehat{x}_3$  as the due care standard for the new activity  $f_3$  and holds the injurer liable to pay damages of  $z_2$  to the victim.<sup>31</sup> This makes the injurers and victims of the world aware of  $f_3$  (and that it is risky). Moreover, they can deduce  $\delta$  from  $\widehat{x}_3$ ; specifically,  $\delta = 2\widehat{x}_3/z_2$ . As a result, they can learn  $\widehat{p}$  and  $\widehat{h}_3(x_3) = \delta z_2 \tau(x_3)$ , without expending additional resources to learn  $\delta$ . Knowledge of  $\widehat{h}_3(x_3)$  is necessary to induce injurers to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of  $z_2$  to the victim. This makes the injurers and victims of the world aware of  $f_3$  (and that it is

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<sup>31</sup>Again, because the world already knows the set of potential harms, our results below would not change if the court does not hold the injurer liable and award damages.

risky). However, they cannot deduce  $\delta$  or learn  $\hat{p}$  or  $\hat{h}_3(x_3)$ . Without knowledge of  $\hat{h}_3(x_3)$ , strict liability cannot induce injurers to take efficient care.

### 4.3 New Consequence

We last consider the case of a new consequence. As with the case of a new act, we assume  $S = Z^F$ :

$p$	$p_1$	$p_2$	$p_3$	$p_4$
$F \setminus S$	$s_1$	$s_2$	$s_3$	$s_4$
$f_1$	0	0	$z_2$	$z_2$
$f_2$	0	$z_2$	0	$z_2$

Given  $S$  and  $p$ , the efficient levels of care are

$$\tilde{x}_1 = \frac{(p_3 + p_4)z_2}{2} \quad \text{and} \quad \tilde{x}_2 = \frac{(p_2 + p_4)z_2}{2}.$$

Under negligence, the court stipulates  $\bar{x}_1 = \tilde{x}_1$  and  $\bar{x}_2 = \tilde{x}_2$  as the due care standards for  $f_1$  and  $f_2$ , respectively.

Suppose the parties discover a new consequence,  $z_3 > z_2$ , which they link to  $f_1$  and  $f_2$ . In particular, suppose that the injurer engages in  $f_1$  and  $f_2$ , that each results in harm  $z_3$ , and that the victim brings a tort suit against the injurer before the court. The feasible state space expands from four to nine states:

$\hat{p}$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$	$\hat{p}_5$	$\hat{p}_6$	$\hat{p}_7$	$\hat{p}_8$	$\hat{p}_9$
$F \setminus \hat{S}$	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$	$s_7$	$s_8$	$s_9$
$f_1$	0	0	$z_2$	$z_2$	$z_3$	$z_3$	0	$z_2$	$z_3$
$f_2$	0	$z_2$	0	$z_2$	0	$z_2$	$z_3$	$z_3$	$z_3$

The expanded feasible state space is characterized by three events, one in which  $f_1$  results in no harm, one in which  $f_1$  results in harm  $z_2$ , and one in which  $f_1$  results in harm  $z_3$ . Each

event contains three states, one in which  $f_2$  results in no harm, one in which  $f_2$  results in  $z_2$ , and one in which  $f_2$  results in  $z_3$ .<sup>32</sup>

We assume that, by virtue of the suit, the parties learn that activity  $f_1$  yields  $z_3$  with probability  $\alpha > 0$  and activity  $f_2$  yields  $z_3$  with probability  $\beta > 0$ . (This is analogous to assuming the parties learn  $\delta$  in the case of a new link or act.) By reverse Bayesianism,

$$\begin{aligned} \frac{p_1}{p_2} &= \frac{\widehat{p}_1}{\widehat{p}_2}, & \frac{p_1}{p_3} &= \frac{\widehat{p}_1}{\widehat{p}_3}, & \frac{p_1}{p_4} &= \frac{\widehat{p}_1}{\widehat{p}_4}, \\ \frac{p_2}{p_3} &= \frac{\widehat{p}_2}{\widehat{p}_3}, & \frac{p_2}{p_4} &= \frac{\widehat{p}_2}{\widehat{p}_4}, & \text{and} & \frac{p_3}{p_4} &= \frac{\widehat{p}_3}{\widehat{p}_4}. \end{aligned}$$

In addition: by definition,  $\alpha = \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9$  and  $\beta = \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9$ ; by the unit measure axiom on  $\widehat{S}$ ,  $\widehat{p}_1 + \dots + \widehat{p}_6 = 1$ ; and by act independence,

$$\begin{aligned} \widehat{p}_5 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5), & \widehat{p}_6 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6), \\ \widehat{p}_7 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_7)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9), & \widehat{p}_8 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9), \\ \text{and } \widehat{p}_9 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9). \end{aligned}$$

It follows that the updated probability measure  $\widehat{p}$  is given by:

**Proposition 3**  $\widehat{p}_1 = (1 - \alpha)(1 - \beta)p_1$ ,  $\widehat{p}_2 = (1 - \alpha)(1 - \beta)p_2$ ,  $\widehat{p}_3 = (1 - \alpha)(1 - \beta)p_3$ ,  
 $\widehat{p}_4 = (1 - \alpha)(1 - \beta)p_4$ ,  $\widehat{p}_5 = \alpha(1 - \beta)(p_1 + p_3)$ ,  $\widehat{p}_6 = \alpha(1 - \beta)(p_2 + p_4)$ ,  
 $\widehat{p}_7 = \beta(1 - \alpha)(p_1 + p_2)$ ,  $\widehat{p}_8 = \beta(1 - \alpha)(p_3 + p_4)$ , and  $\widehat{p}_9 = \alpha\beta$ .

Note that the degree of unawareness is  $\delta = \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9 = \alpha + \beta - \alpha\beta$  and that  $1 - \delta = (1 - \alpha)(1 - \beta)$ . We can rewrite  $\widehat{p}$  in terms of  $\delta$  as follows:

**Corollary 1**  $\widehat{p}_1 = (1 - \delta)p_1$ ,  $\widehat{p}_2 = (1 - \delta)p_2$ ,  $\widehat{p}_3 = (1 - \delta)p_3$ ,  $\widehat{p}_4 = (1 - \delta)p_4$ ,  
 $\widehat{p}_5 = \frac{\alpha}{1 - \alpha}(1 - \delta)(p_1 + p_3) = (\delta - \beta)(p_1 + p_3)$ ,  $\widehat{p}_6 = \frac{\alpha}{1 - \alpha}(1 - \delta)(p_2 + p_4) = (\delta - \beta)(p_2 + p_4)$ ,  
 $\widehat{p}_7 = \frac{\beta}{1 - \beta}(1 - \delta)(p_1 + p_2) = (\delta - \alpha)(p_1 + p_2)$ ,  $\widehat{p}_8 = \frac{\beta}{1 - \beta}(1 - \delta)(p_3 + p_4) = (\delta - \alpha)(p_3 + p_4)$ ,  
and  $\widehat{p}_9 = \alpha + \beta - \delta$ .

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<sup>32</sup>Note that the conceivable state space also expands from four to nine states, so  $\widehat{S} = \widehat{Z}^F$ .

Given  $\widehat{S}$  and  $\widehat{p}$ , the efficient levels of care are

$$\begin{aligned}\widehat{x}_1 &= \frac{(\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8)z_2 + (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)z_3}{2} = \frac{(1 - \alpha)(p_3 + p_4)z_2 + \alpha z_3}{2} \\ \text{and } \widehat{x}_2 &= \frac{(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6)z_2 + (\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9)z_3}{2} = \frac{(1 - \beta)(p_2 + p_4)z_2 + \beta z_3}{2}.\end{aligned}$$

Note that  $\widehat{x}_1 > \widetilde{x}_1$  and  $\widehat{x}_2 > \widetilde{x}_2$ . Thus, the discovery of  $z_3$  necessitates the stipulation of new due care standards for both  $f_1$  and  $f_2$ .

Under negligence, the court stipulates  $\widehat{x}_1 = \widetilde{x}_1$  and  $\widehat{x}_2 = \widetilde{x}_2$  as the new due care standards for  $f_1$  and  $f_2$ , respectively. The court holds the injurer liable to pay damages of  $z_3$  to the victim with respect to each of  $f_1$  and  $f_2$ . This makes the injurers and victims of the world aware of  $z_3$  (and that it is linked to  $f_1$  and  $f_2$ ).<sup>33</sup> Moreover, they can deduce  $\alpha$  and  $\beta$  (and, therefore,  $\delta$ ) from  $\widehat{x}_1$  and  $\widehat{x}_2$ ; specifically,

$$\alpha = \frac{p_3 z_2 - 2\widehat{x}_1 + p_4 z_2}{p_3 z_2 - z_3 + p_4 z_2} \quad \text{and} \quad \beta = \frac{p_2 z_2 - 2\widehat{x}_2 + p_4 z_2}{p_2 z_2 - z_3 + p_4 z_2}.$$

As a result, they can learn  $\widehat{p}$  and

$$\begin{aligned}\widehat{h}_1(x_1) &= [(1 - \alpha)(p_3 + p_4)z_2 + \alpha z_3]\tau(x_1) \\ \text{and } \widehat{h}_2(x_2) &= [(1 - \beta)(p_2 + p_4)z_2 + \beta z_3]\tau(x_2),\end{aligned}$$

without expending additional resources to learn  $\alpha$  and  $\beta$ . Knowledge of  $\widehat{h}_1(x_1)$  and  $\widehat{h}_2(x_2)$  is necessary to induce injurers to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of  $z_3$  for each instance of harm. This makes the injurers and victims of the world aware of  $z_3$  (and that it is linked to  $f_1$  and  $f_2$ ). However, they cannot deduce  $\alpha$  or  $\beta$  or learn  $\widehat{p}$ ,  $\widehat{h}_1(x_1)$ , or  $\widehat{h}_2(x_2)$ . Without knowledge of  $\widehat{h}_1(x_1)$  and  $\widehat{h}_2(x_2)$ , strict liability cannot induce efficient care.

<sup>33</sup>Even if the court does not hold the injurer liable and award damages, the victim's claims make the world aware of  $z_3$  (and that it is linked to  $f_1$  and  $f_2$ ).

## 4.4 Act Independence

Before turning to the general results, we conclude this section with a few remarks about the act independence assumption and the extent to which it drives our results.

As revealed by the proofs of Propositions 1 through 3, reverse Bayesianism alone is not sufficient to fully determine the updated probability distribution  $\hat{p}$ . To borrow a term from the econometrics literature, reverse Bayesianism only partially identifies  $\hat{p}$ . The reason is that reverse Bayesianism implies restrictions on the updated probabilities of nonnull states in the original state space (or, in the case of a new act, their corresponding events in the expanded state space), but not on the probabilities of new states in the expanded state space. In other words, reverse Bayesianism prescribes how probability mass shifts away from nonnull states in the original state space to the corresponding states or events in the expanded state space, but it does not dictate how the shifted probability mass is distributed among the new states in the expanded state space.<sup>34</sup> This is where act independence comes in. It determines how the shifted probability mass is apportioned among the new states in the expanded state space. Together, reverse Bayesianism and act independence fully identify  $\hat{p}$ .

How reasonable is the act independence assumption? The answer, of course, depends on the nature of the specific activities in question. To be sure, there exist activities whose outcomes are not independent. It is useful, therefore, to investigate the importance of the act independence assumption for our results.

**New link.** In the case of a new link, reverse Bayesianism alone implies  $\hat{p}_1 = (1 - \delta)p_1$ ,  $\hat{p}_2 = (1 - \delta)p_2$ , and  $\hat{p}_3 + \hat{p}_4 = \delta$ . Importantly, reverse Bayesianism alone is not sufficient to separately identify  $\hat{p}_3$  and  $\hat{p}_4$ . As it turns out, this does not create an issue with respect to activity  $f_1$ . Recall that, by assumption, the parties learn  $\delta$  (the probability that  $f_1$  yields  $z_2$ ). Because the efficient level of care for  $f_1$  is a function of the sum  $\hat{p}_3 + \hat{p}_4$ , the court can

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<sup>34</sup>Karni and Vierø (2013, p. 2805) highlight this feature of reverse Bayesianism in their concluding remarks: "The model presented in this article predicts that, as awareness grows and the state space expands, the relative likelihoods of events in the original state space remain unchanged. The model is silent about the absolute levels of these probabilities. In other words, our theory does not predict the probability of the new events in the expanded state space."

stipulate a new due care standard for  $f_1$  in terms of  $\delta$ . This is sufficient to make the injurers and victims of the world aware that  $f_1$  is risky. Moreover, they can deduce  $\delta$  from the new due care standard for  $f_1$  and, in turn, learn  $\widehat{h}_1(x_1) = \delta z_2 \tau(x_1)$ .

Relaxing act independence, however, creates ambiguity with respect to the updated risk of activity  $f_2$ . Because the efficient level of care for  $f_2$  is a function of the sum  $\widehat{p}_2 + \widehat{p}_4$ , without act independence (or another assumption that separately identifies  $\widehat{p}_3$  and  $\widehat{p}_4$ ), the court cannot stipulate a precise new due care standard for  $f_2$ . The best the court can do is specify lower and upper bounds, using the knowledge that  $\widehat{p}_4 \in (0, \delta)$ . Given these bounds, the best the injurers and victims of the world can do is infer bounds on  $\widehat{h}_2(x_2)$ .

Of course, the ambiguity can be resolved if, by virtue of the suit, the parties learn more about  $\widehat{p}$ . For instance, if the parties learn not only  $\delta$  but also either  $\widehat{p}_2 + \widehat{p}_4$  (the updated probability that  $f_2$  yields  $z_2$ ) or  $\widehat{p}_4$  (the joint probability that  $f_1$  and  $f_2$  yield  $z_2$ ), this is sufficient to separately identify  $\widehat{p}_3$  and  $\widehat{p}_4$ . With this, the court can stipulate a precise new due care standard for  $f_2$ , from which the injurers and victims of the world can deduce  $\widehat{p}_2 + \widehat{p}_4$  and, in turn, learn  $\widehat{h}_2(x_2) = (\widehat{p}_2 + \widehat{p}_4) z_2 \tau(x_2)$ .

**New act.** In the case of a new act, reverse Bayesianism alone implies  $\widehat{p}_1 + \widehat{p}_5 = p_1$ ,  $\widehat{p}_2 + \widehat{p}_6 = p_2$ ,  $\widehat{p}_3 + \widehat{p}_7 = p_3$ ,  $\widehat{p}_4 + \widehat{p}_8 = p_4$ , and  $\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 = \delta$ . Recall that the efficient level of care for  $f_1$  is a function of the sum  $\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8$ ; the efficient level of care for  $f_2$  is a function of the sum  $\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8$ ; and the efficient level of care for  $f_3$  is a function of the sum  $\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8$ . Hence, even without act independence, the court's information is sufficiently precise (i) to know that it need not stipulate new due care standards for activities  $f_1$  and  $f_2$  and (ii) to stipulate a due care standard for the new activity  $f_3$ . This makes the injurers and victims of the world aware of  $f_3$  (and that it is risky). Moreover, they can deduce  $\delta$  from the due care standard for  $f_3$  and, in turn, learn  $\widehat{h}_3(x_3) = \delta z_2 \tau(x_3)$ .

**New consequence.** In the case of a new consequence, reverse Bayesianism alone implies  $\widehat{p}_1 = (1 - \delta)p_1$ ,  $\widehat{p}_2 = (1 - \delta)p_2$ ,  $\widehat{p}_3 = (1 - \delta)p_3$ ,  $\widehat{p}_4 = (1 - \delta)p_4$ , and  $\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9 = \delta$ . By assumption, the parties learn  $\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9 = \alpha$  (the probability that  $f_1$  yields  $z_3$ ) and

$\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9 = \beta$  (the probability that  $f_2$  yields  $z_3$ ). Assume the parties also learn  $\widehat{p}_9$  (the joint probability that  $f_1$  and  $f_2$  yield  $z_3$ ), and let  $\widehat{p}_9 = \gamma$ . Note that  $\delta = \alpha + \beta - \gamma$ .

Recall that the efficient level of care for activity  $f_1$  is a function of  $\alpha$  and the sum  $\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8$  (the updated probability that  $f_1$  yields  $z_2$ ), and the efficient level of care for activity  $f_2$  is a function of  $\beta$  and the sum  $\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6$  (the updated probability that  $f_2$  yields  $z_2$ ). Without act independence, the sums  $\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8$  and  $\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6$  are only partially identified (because  $\widehat{p}_6$  and  $\widehat{p}_8$  are not separately identified), creating ambiguity with respect to the updated risks of both activities. As a result, the court cannot stipulate precise new due care standards for activities  $f_1$  and  $f_2$ . The best the court can do is specify lower and upper bounds:

$$\begin{aligned} \widehat{x}_1 &\in \left( \frac{(1-\delta)(p_3+p_4)z_2 + \alpha z_3}{2}, \frac{((1-\delta)(p_3+p_4) + \delta)z_2 + \alpha z_3}{2} \right) \\ \text{and } \widehat{x}_2 &\in \left( \frac{(1-\delta)(p_2+p_4)z_2 + \beta z_3}{2}, \frac{((1-\delta)(p_2+p_4) + \delta)z_2 + \beta z_3}{2} \right). \end{aligned}$$

Given these bounds, and given that the victim's claims make the world aware of  $z_3$  (and that it is linked to  $f_1$  and  $f_2$ ), the injurers and victims of the world can deduce  $\alpha$ ,  $\beta$ , and  $\delta$ ; however, the best they can do is infer bounds on  $\widehat{h}_1(x_1)$  and  $\widehat{h}_2(x_2)$ .

As before, the ambiguity can be resolved if, by virtue of the suit, the parties learn more about  $\widehat{p}$ . For instance, if the parties learn not only  $\delta$  and  $\gamma$  but also either  $\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8$  or  $\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6$ , this is sufficient to separately identify  $\widehat{p}_5$ ,  $\widehat{p}_6$ ,  $\widehat{p}_7$ , and  $\widehat{p}_8$ . With this, the court can stipulate precise new due care standards for  $f_1$  and  $f_2$ , from which the injurers and victims of the world can learn  $\widehat{h}_1(x_2)$  and  $\widehat{h}_2(x_2)$ .

In summary, without act independence, reverse Bayesianism only partially identifies  $\widehat{p}$ . This does not create an issue in the case of a new act—the court's information is sufficiently precise to stipulate a due care standard with respect to each activity. In the case of a new link or consequence, however, the partial identification of  $\widehat{p}$  creates ambiguity with respect to the updated risk of one or both activities, leading to imprecise due care standards. In short, we might say that, without act independence, negligence achieves only "boundedly

optimal" deterrence. That said, negligence still has a partial advantage over strict liability. What's more, the ambiguity in any case can be resolved if the parties learn more about  $\hat{p}$ . In other words, the more the parties learn about the updated probability of harm, the less important is the act independence assumption for our results.

## 5 General Results

In this section, we show that our results extend to a more general model with  $m$  acts and  $n$  consequences. We also relax the shape restrictions on the care cost and expected harm reduction functions and assume only that each is convex.

Let  $F = \{f_1, \dots, f_m\}$  be the set of activities and  $Z = \{z_1, \dots, z_n\}$  be the set of harms, where  $0 \leq z_1 < z_2 < \dots < z_n$ . For each activity  $f_i$ , the cost of taking care  $x_i \geq 0$  is  $c(x_i)$ , where  $c(0) = 0$ ,  $c'(x_i) > 0$ , and  $c''(x_i) > 0$  for all  $x_i \geq 0$ . Activity  $f_i$ 's expected harm is  $h_i(x_i) \equiv \sum_{j=1}^n \pi_{ij} z_j \tau(x_i)$ , where (i)  $\pi_{ij}$  is the probability that  $f_i$  causes  $z_j$  and (ii)  $\tau(x_i) \in (0, 1]$ ,  $\tau(0) = 1$ ,  $\tau'(x_i) < 0$ , and  $\tau''(x_i) \geq 0$  for all  $x_i \geq 0$ .<sup>35</sup>

Given  $F$  and  $Z$ , the conceivable state space is  $Z^F$ , where each state  $s \in Z^F$  is a vector of length  $m$ , the  $i$ th element of which,  $s^i$ , is the harm  $z_j \in Z$  caused by activity  $f_i \in F$  in that state. The feasible state space is  $S \equiv Z^F \setminus N$ , where  $N \subset Z^F$  is the set of null states. Each state in  $N$  is induced by a nullified link between an activity  $f_i$  and a harm  $z_j$ .

Let  $p$  represent the parties' common beliefs on  $Z^F$ . The support set of  $p$  is  $S$ . That is,  $p(s) > 0$  for all  $s \in S$  and  $p(s) = 0$  for all  $s \in N$ .

Given  $S$  and  $p$ , the efficient levels of care are  $\tilde{x}_i = \xi^{-1} \left( \sum_{j=1}^n \pi_{ij} z_j \right)$ ,  $i = 1, \dots, m$ , where (i)  $\xi^{-1}$  denotes the inverse of  $\xi(x_i) \equiv -c'(x_i)/\tau'(x_i)$  and (ii)  $\pi_{ij} = \sum_{s \in S: s^i = z_j} p(s)$ . Under negligence, the court stipulates  $\bar{x}_i = \tilde{x}_i$  as the due care standard for each activity  $f_i$ .

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<sup>35</sup>For example, we could have  $\tau(x_i) = e^{-x_i}$ .

## 5.1 New Link

Assume  $N \neq \emptyset$ , so  $S \subset Z^F$ . Suppose the parties discover a new link from  $f_l$  to  $z_k$  for some  $l \in \{1, \dots, m\}$  and  $k \in \{1, \dots, n\}$ . Let  $\widehat{S}$  denote the expanded feasible state space and  $\widehat{p}$  denote the parties' updated beliefs. In addition, let  $\Delta = \widehat{S} \setminus S$ . We assume that, by virtue of a tort litigation, the parties learn that  $f_l$  yields  $z_k$  with probability  $\delta > 0$ .

By reverse Bayesianism,  $p(s)/p(t) = \widehat{p}(s)/\widehat{p}(t)$  for all  $s, t \in S$ . In addition,  $\delta = \widehat{p}(\Delta)$  by definition and  $\sum_{s \in \widehat{S}} \widehat{p}(s) = 1$  by the unit measure axiom on  $\widehat{S}$ . Moreover, by act independence,  $\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i))$  for all  $s = (s^1, \dots, s^m) \in \Delta$ , where  $A_i(z_j) \equiv \{t \in \widehat{S} : t^i = z_j\}$  is the event that activity  $f_i$  yields harm  $z_j$ .

Given any  $s \in \Delta$ , let  $L(s) \equiv \{t \in S : t^i = s^i, \forall i \neq l\}$  denote the event in  $S$  that corresponds to  $s \in \Delta$ . It follows that:

**Proposition 4** *In the case of a new link involving  $f_l$ :*

(i)  $\widehat{p}(s) = (1 - \delta)p(s)$  for all  $s \in S$ ; and

(ii)  $\widehat{p}(s) = \delta p(L(s))$  for all  $s \in \Delta$ .

Given  $\widehat{S}$  and  $\widehat{p}$ , the efficient levels of care are  $\widehat{x}_i = \xi^{-1} \left( \sum_{j=1}^n \widehat{\pi}_{ij} z_j \right)$ ,  $i = 1, \dots, m$ , where  $\widehat{\pi}_{ij} = \sum_{s \in \widehat{S} : s^i = z_j} \widehat{p}(s)$ . Specifically:

**Proposition 5** *In the case of a new link from  $f_l$  to  $z_k$ :*

(i)  $\widehat{x}_l = \xi^{-1} \left( \sum_{j=1}^n (1 - \delta) \pi_{lj} z_j + \delta z_k \right)$ ; and

(ii)  $\widehat{x}_i = \widetilde{x}_i$  for all  $i \neq l$ .

**Corollary 2**  $\widehat{x}_l = \widetilde{x}_l$  if and only if  $z_k = \sum_{j=1}^n \pi_{lj} z_j$ .

Thus, the discovery that  $f_l$  can yield  $z_k$  necessitates the stipulation of a new due care standard for  $f_l$  (unless  $z_k = \sum_{j=1}^n \pi_{lj} z_j$ ) but not for the other activities.

Under negligence, the court stipulates  $\widehat{x}_l = \widehat{\widetilde{x}}_l$  as the new due care standard for  $f_l$  (or restipulates  $\widehat{x}_l = \widetilde{x}_l$  if  $z_k = \sum_{j=1}^n \pi_{lj} z_j$ ) and holds the injurer liable to pay damages of  $z_k$  to the victim if  $\widehat{x}_l > \widetilde{x}_l$ . This, along with the victim's claim, makes the injurers and victims of the world aware that  $f_l$  can yield  $z_k$ . Moreover, they can deduce  $\delta$  from  $\widehat{\widetilde{x}}_l$ .

**Proposition 6** *In the case of a new link from  $f_l$  to  $z_k$ ,  $\delta = \frac{c'(\widehat{x}_l) + \sum_{j=1}^n \pi_{lj} z_j \tau'(\widehat{x}_l)}{\sum_{j=1}^n \pi_{lj} z_j \tau'(\widehat{x}_l) - z_k \tau'(\widehat{x}_l)}$ .*

As a result, they can learn  $\widehat{p}$  and  $\widehat{h}_l(x_l) = \sum_{j=1}^n [(1 - \delta)\pi_{lj} z_j + \delta z_k] \tau(x_l)$ , without expending additional resources to learn  $\delta$ . Knowledge of  $\widehat{h}_l(x_l)$  is necessary to induce injurers to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of  $z_k$  to the victim. This makes the injurers and victims of the world aware that  $f_l$  can yield  $z_k$ . However, they cannot deduce  $\delta$  or learn  $\widehat{p}$  or  $\widehat{h}_l(x_l)$ . Without knowledge of  $\widehat{h}_l(x_l)$ , strict liability cannot induce injurers to take efficient care.

## 5.2 New Act

Assume  $S \subseteq Z^F$ . Suppose the parties discover a new act,  $f_{m+1}$ . Let  $\widehat{S}$  denote the expanded feasible state space and  $\widehat{p}$  denote the parties' updated beliefs. We assume that, by virtue of a tort litigation, the parties learn that  $f_{m+1}$  yields  $z_j$  with probability  $\delta_j > 0$  for all  $j = 1, \dots, n$ .<sup>36</sup> Note that  $\sum_{j=1}^n \delta_j = 1$ .

By reverse Bayesianism,  $p(s)/p(t) = \widehat{p}(E(s))/\widehat{p}(E(t))$  for all  $s, t \in S$ , where  $E(s) \equiv \{t \in \widehat{S} : t^i = s^i, \forall i \neq m+1\}$  denotes the event in  $\widehat{S}$  that corresponds to  $s \in S$ . Note that  $\{E(s) : s \in S\}$  forms a partition of  $\widehat{S}$  and that  $|E(s)| = n$  for all  $s \in S$ . With a slight abuse of notation, index the states in each  $E(s)$  by  $j = 1, \dots, n$ .

By definition,  $\delta_j = \widehat{p}(A_{m+1}(z_j))$ , where  $A_i(z_j) \equiv \{t \in \widehat{S} : t^i = z_j\}$  is the event that activity  $f_i$  yields harm  $z_j$ . In addition,  $\sum_{s \in \widehat{S}} \widehat{p}(s) = 1$  by the unit measure axiom on  $\widehat{S}$ . Moreover, by act independence,  $\widehat{p}(s) = \prod_{i=1}^{m+1} \widehat{p}(A_i(s^i))$  for all  $s = (s^1, \dots, s^{m+1}) \in \widehat{S}$ .

It follows that:

**Proposition 7** *In the case of a new act  $f_{m+1}$ , for all  $s \in S$  and corresponding  $E(s) \subset \widehat{S}$ ,  $\widehat{p}(s_j) = \delta_j p(s)$  for all  $s_j \in E(s)$ ,  $j = 1, \dots, n$ .*

<sup>36</sup> Assuming  $\delta_j > 0$  for all  $j = 1, \dots, n$  is without loss of generality. We can deal with the case where  $\delta_j = 0$  for some  $j$  by assuming  $\delta_j > 0$  for the first  $k < n$  and changing  $n$  to  $k$  as necessary in the statements below.

Given  $\widehat{S}$  and  $\widehat{p}$ , the efficient levels of care are  $\widehat{x}_i = \xi^{-1} \left( \sum_{j=1}^n \widehat{\pi}_{ij} z_j \right)$ ,  $i = 1, \dots, m+1$ , where  $\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s)$ . Specifically:

**Proposition 8** *In the case of a new act  $f_{m+1}$ :*

- (i)  $\widehat{x}_i = \widetilde{x}_i$  for all  $i \neq m+1$ ; and
- (ii)  $\widehat{x}_{m+1} = \xi^{-1} \left( \sum_{j=1}^n \delta_j z_j \right)$ .

Thus, the discovery of  $f_{m+1}$  necessitates the stipulation of a new due care standard,  $\widehat{x}_{m+1}$ , but it does not necessitate the stipulation of new due care standards for  $f_1, \dots, f_m$ .

Under negligence, the court stipulates  $\widehat{x}_{m+1} = \widetilde{x}_{m+1}$  as the due care standard for the new activity  $f_{m+1}$  and holds the injurer liable to pay damages to the victim. This makes the injurers and victims of the world aware of  $f_{m+1}$  (and that it is risky). Although they cannot separately deduce each  $\delta_j$  from  $\widehat{x}_{m+1}$ ,<sup>37</sup> they nevertheless can infer  $\widehat{h}_{m+1}(x_{m+1})$  from  $\widehat{x}_{m+1}$ , without expending additional resources to learn all  $\delta_j$ .

**Proposition 9** *In the case of a new act  $f_{m+1}$ ,  $\widehat{h}_{m+1}(x_{m+1}) = -\frac{c'(\widehat{x}_{m+1})}{\tau'(\widehat{x}_{m+1})} \tau(x_{m+1})$ .*

Knowledge of  $\widehat{h}_{m+1}(x_{m+1})$  is necessary to induce the world's injurers to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages to the victim. This makes the injurers and victims of the world aware of  $f_{m+1}$  (and that it is risky). However, they do not learn  $\widehat{h}_{m+1}(x_{m+1})$ . Without knowledge of  $\widehat{h}_{m+1}(x_{m+1})$ , strict liability cannot induce injurers to take efficient care.

### 5.3 New Consequence

Assume  $S \subseteq Z^F$ . Suppose the parties discover a new consequence,  $z_{n+1}$ . Let  $\widehat{S}$  denote the expanded feasible state space and  $\widehat{p}$  denote the parties' updated beliefs. In addition, let  $\Delta = \widehat{S} \setminus S$  and  $\delta = \widehat{p}(\Delta)$ . We assume that, by virtue of a tort litigation, the parties learn that  $f_i$  yields  $z_{n+1}$  with probability  $\alpha_i > 0$  for all  $i = 1, \dots, m$ .<sup>38</sup> Note that  $1 - \delta = \prod_{i=1}^m (1 - \alpha_i)$ .

<sup>37</sup>Note, however, that if each  $z_j$  is a different type of harm that requires a different type of care, then the court would stipulate a different due care standard  $\widehat{x}_{m+1,j}$  with respect to each  $z_j$ , in which case the injurers and victims of the world could separately deduce each  $\delta_j$ .

<sup>38</sup>Assuming  $\alpha_i > 0$  for all  $i$  is without loss of generality. We can deal with the case where  $\alpha_i > 0$  for some  $i$  by assuming  $\alpha_i > 0$  for the first  $l < m$  and changing  $m$  to  $l$  as necessary in the statements below.

By reverse Bayesianism,  $p(s)/p(t) = \widehat{p}(s)/\widehat{p}(t)$  for all  $s, t \in S$ . In addition,  $\alpha_i = \widehat{p}(A_i(z_{n+1}))$  by definition and  $\sum_{s \in \widehat{S}} \widehat{p}(s) = 1$  by the unit measure axiom on  $\widehat{S}$ . Moreover, by act independence,  $\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i))$  for all  $s = (s^1, \dots, s^m) \in \Delta$ .

Given any  $s \in \Delta$ , let  $I(s) \equiv \{i \in \{1, \dots, m\} : s^i = z_{n+1}\}$  denote the indices of the acts that yield  $z_{n+1}$  in that state of the world, let  $\bar{I}(s) \equiv \{i \in \{1, \dots, m\} : s^i \neq z_{n+1}\}$  denote the indices of the acts that do not yield  $z_{n+1}$  in that state of the world, and let  $C(s) \equiv \{t \in S : t^i = s^i, \forall i \in \bar{I}(s)\}$  denote the event in  $S$  that corresponds to  $s \in \Delta$  on  $\bar{I}(s)$ .

It follows that:

**Proposition 10** *In the case of a new consequence  $z_{n+1}$ :*

- (i)  $\widehat{p}(s) = (\prod_{i=1}^m (1 - \alpha_i)) p(s)$  for all  $s \in S$ ;
- (ii)  $\widehat{p}(s) = \left( \prod_{i \in I(s)} \alpha_i \right) \left( \prod_{i \in \bar{I}(s)} (1 - \alpha_i) \right) p(C(s))$  for all  $s \in \Delta$  such that  $I(s) \subset \{1, \dots, m\}$ ;
- (iii)  $\widehat{p}(s) = \prod_{i=1}^m \alpha_i$  for the  $s \in \Delta$  such that  $I(s) = \{1, \dots, m\}$ .

Given  $\widehat{S}$  and  $\widehat{p}$ , the efficient levels of care are  $\widehat{x}_i = \xi^{-1} \left( \sum_{j=1}^{n+1} \widehat{\pi}_{ij} z_j \right)$ ,  $i = 1, \dots, m$ , where  $\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s)$ . Specifically:

**Proposition 11** *In the case of a new consequence  $z_{n+1}$ ,  $\widehat{x}_i = \xi^{-1} \left( \sum_{j=1}^n (1 - \alpha_i) \pi_{ij} z_j + \alpha_i z_{n+1} \right)$  for all  $i = 1, \dots, m$ .*

**Corollary 3**  $\widehat{x}_i = \widetilde{x}_i$  if and only if  $z_{n+1} = \sum_{j=1}^n \pi_{ij} z_j$ .

Thus, the discovery of  $z_{n+1}$  necessitates the stipulation of new due care standards for each activity  $f_i$  such that  $z_{n+1} \neq \sum_{j=1}^n \pi_{ij} z_j$ .

Under negligence, the court stipulates  $\widehat{x}_i = \widetilde{x}_i$ ,  $i = 1, \dots, m$ , as the new due care standards for  $f_1, \dots, f_m$  (or restipulates  $\widehat{x}_i = \widetilde{x}_i$  if  $z_{n+1} = \sum_{j=1}^n \pi_{ij} z_j$ ) and holds the injurer liable to pay damages of  $z_{n+1}$  to the victim with respect to each activity  $f_i$  such that  $\widehat{x}_i > \widetilde{x}_i$ . This, along with the victim's claims, makes the injurers and victims of the world aware of  $z_{n+1}$  (and that it is linked to  $f_1, \dots, f_m$ ). Moreover, they can deduce  $\alpha_1, \dots, \alpha_m$  from  $\widehat{x}_1, \dots, \widehat{x}_m$ .

**Proposition 12** *In the case of a new consequence  $z_{n+1}$ ,  $\alpha_i = \frac{c'(\widehat{x}_i) + \sum_{j=1}^n \pi_{ij} z_j \tau'(\widehat{x}_i)}{\sum_{j=1}^n \pi_{ij} z_j \tau'(\widehat{x}_i) - z_{n+1} \tau'(\widehat{x}_i)}$  for all  $i = 1, \dots, m$ .*

As a result, they can learn  $\hat{p}$  and  $\hat{h}_1(x_1), \dots, \hat{h}_m(x_m)$ , without expending additional resources to learn  $\alpha_1, \dots, \alpha_m$ . Knowledge of  $\hat{h}_1(x_1), \dots, \hat{h}_m(x_m)$  is necessary to induce efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of  $z_{n+1}$  to the victim with respect to each activity  $f_i$ . This makes the injurers and victims of the world aware of  $z_{n+1}$  (and that it is linked to  $f_1, \dots, f_m$ ). However, they can not deduce  $\alpha_1, \dots, \alpha_m$  or learn  $\hat{p}$  or  $\hat{h}_1(x_1), \dots, \hat{h}_m(x_m)$ . Again, without knowledge of  $\hat{h}_1(x_1), \dots, \hat{h}_m(x_m)$ , strict liability cannot induce efficient care.

## 6 Discussion

We compare and contrast negligence and strict liability in a world with unawareness, and find that negligence has a key advantage—the due care standard serves as a knowledge transfer mechanism. Under either tort liability rule, a suit involving a newly discovered act, consequence, or act-consequence link makes the world aware of a new possibility of harm. But only negligence, through the stipulation of new due care standards, spreads awareness about the updated probability of harm. As such, the negligence due care standard is like a public good. The social benefit of spreading awareness about the updated probability of harm is that the injurers and victims of the world need not expend additional resources to develop this knowledge—wastefully duplicative activity that would be necessary to achieve optimal deterrence under strict liability. In a sense, negligence is akin to patents; both carry social costs (negligence is more costly to administer; patents create monopolies and deadweight loss), yet both provide social benefits in terms of knowledge transmission.

To model unawareness and growing awareness, we adopt the reverse Bayesian approach of Karni and Vierø (2013). The reverse Bayesian model has (at least) two attractive features. The first is transparency. Karni and Vierø provide an axiomatic foundation for the model, and so one can judge the theory by the axioms.<sup>39</sup> The second attractive feature of the model is its accessibility. The model is built upon a familiar choice-theoretic framework

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<sup>39</sup>The key axioms of the model are the "consistency" axioms, which essentially require that preferences conditional on the original state of awareness are not altered by growing awareness.

(subjective expected utility theory), and the upshot is a belief revision theory that mirrors the process of Bayesian updating.<sup>40</sup> At the same time, the model has its shortcomings. For instance, Chambers and Hayashi (2018) criticize its empirical content from a revealed preference perspective. They show that, in the case of a new consequence, the model does not make singular predictions about observable choices over feasible acts. A second shortcoming of the model is that it assumes a naive or myopic unawareness—people are unaware that they are unaware. A sophisticated unawareness, where people are aware that they are unaware, may be more realistic. Aware of this shortcoming, Karni and Vierø (2017) extend their model to the case of sophisticated unawareness. The end result is a generalization that maintains the flavor of reverse Bayesianism and nests the naive model as a special case.

The pros and cons of the model aside, one might question the importance of our results in a world with safety regulation in addition to tort liability. In such a world, one could argue, there are regulators and other non-court actors who can spread awareness about newly discovered risks. While this may be correct, it is orthogonal to our inquiry. We are contributing to the negligence versus strict liability debate in tort law. We therefore consider a world where the tort system is the only mechanism for regulating risky activities, and we compare and contrast the two primary tort liability rules. If one were to consider a world with safety regulation in addition to tort law, one would have to wade into the liability versus regulation debate (e.g., Shavell, 1984a,b; Posner, 2010), and conclude that regulation is the more efficient method of social control, before one could assert that the possibility of safety regulation renders moot the debate over tort liability rules.

The importance of unawareness and growing awareness—via technological progress, scientific discovery, or otherwise—plainly extends beyond the case of unilateral accidents with fixed activity levels. Natural extensions of this paper, therefore, would entail introducing unawareness into other accident settings. In addition, future research could examine the implications of unawareness for the economic analysis of other areas of law such as contract remedies and criminal law or of other legal topics such as litigation and settlement.

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<sup>40</sup>This feature prompts Dominiak and Tserenjigmid (2018, p. 3) to describe the model as "elegant."

# Appendix

## Proof of Proposition 1

By reverse Bayesianism, the definition of  $\delta$ , and the unit measure axiom on  $\widehat{S}$ , we have two linearly independent equations,

$$\widehat{p}_2 = \frac{p_2}{p_1}\widehat{p}_1 \quad \text{and} \quad \widehat{p}_1 + \widehat{p}_2 = 1 - \delta,$$

and two unknowns,  $\widehat{p}_1$  and  $\widehat{p}_2$ . Substituting the first equation into the second, we have

$$\widehat{p}_1 + \frac{p_2}{p_1}\widehat{p}_1 = 1 - \delta,$$

which implies

$$\widehat{p}_1 = \frac{(1 - \delta)p_1}{p_1 + p_2} = (1 - \delta)p_1,$$

where the last equality follows from the unit measure axiom on  $S$  (which implies  $p_1 + p_2 = 1$ ).

It follows that

$$\widehat{p}_2 = \frac{p_2}{p_1}(1 - \delta)p_1 = (1 - \delta)p_2.$$

By act independence and the definition of  $\delta$ , we have

$$\widehat{p}_3 = \delta(\widehat{p}_1 + \widehat{p}_3) \quad \text{and} \quad \widehat{p}_4 = \delta(\widehat{p}_2 + \widehat{p}_4),$$

which imply

$$\widehat{p}_3 = \frac{\delta}{1 - \delta}\widehat{p}_1 \quad \text{and} \quad \widehat{p}_4 = \frac{\delta}{1 - \delta}\widehat{p}_2.$$

It follows that

$$\widehat{p}_3 = \frac{\delta}{1 - \delta}(1 - \delta)p_1 = \delta p_1 \quad \text{and} \quad \widehat{p}_4 = \frac{\delta}{1 - \delta}(1 - \delta)p_2 = \delta p_2.$$

## Proof of Proposition 2

Reverse Bayesianism implies three linearly independent conditions:<sup>41</sup>

$$\begin{aligned} p_2(\widehat{p}_1 + \widehat{p}_5) &= p_1(\widehat{p}_2 + \widehat{p}_6), \\ p_3(\widehat{p}_1 + \widehat{p}_5) &= p_1(\widehat{p}_3 + \widehat{p}_7), \\ \text{and } p_4(\widehat{p}_1 + \widehat{p}_5) &= p_1(\widehat{p}_4 + \widehat{p}_8). \end{aligned}$$

Summing the left- and right-hand sides, and adding  $p_1(\widehat{p}_1 + \widehat{p}_5)$  to each side, yields

$$(p_1 + p_2 + p_3 + p_4)(\widehat{p}_1 + \widehat{p}_5) = (\widehat{p}_1 + \cdots + \widehat{p}_8)p_1$$

By the unit measure axioms on  $S$  and  $\widehat{S}$ , we have  $\widehat{p}_1 + \widehat{p}_5 = p_1$ . Substituting this back into the reverse Bayesian conditions yields

$$\widehat{p}_1 + \widehat{p}_5 = p_1, \quad \widehat{p}_2 + \widehat{p}_6 = p_2, \quad \widehat{p}_3 + \widehat{p}_7 = p_3, \quad \text{and} \quad \widehat{p}_4 + \widehat{p}_8 = p_4.$$

By act independence and the definition of  $\delta$ , we have

$$\begin{aligned} \widehat{p}_5 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_1 + \widehat{p}_5)\delta, \\ \widehat{p}_6 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_2 + \widehat{p}_6)\delta, \\ \widehat{p}_7 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_3 + \widehat{p}_7)\delta, \\ \text{and } \widehat{p}_8 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_4 + \widehat{p}_8)\delta, \end{aligned}$$

where the second equality follows from iterative application of act independence.<sup>42</sup> These imply

$$\widehat{p}_5 = \frac{\delta}{1 - \delta}\widehat{p}_1, \quad \widehat{p}_6 = \frac{\delta}{1 - \delta}\widehat{p}_2, \quad \widehat{p}_7 = \frac{\delta}{1 - \delta}\widehat{p}_3, \quad \text{and} \quad \widehat{p}_8 = \frac{\delta}{1 - \delta}\widehat{p}_4.$$

<sup>41</sup>Note that the six reverse Bayesianism conditions are not linearly independent. In particular, we can derive the last three conditions from the first three.

<sup>42</sup>For example,  $(\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7) = \widehat{p}(\{s_1, s_2, s_5, s_6\})\widehat{p}(\{s_1, s_3, s_5, s_7\}) = \widehat{p}(\{s_1, s_2, s_5, s_6\} \cap \{s_1, s_3, s_5, s_7\}) = \widehat{p}(\{s_1, s_5\}) = \widehat{p}_1 + \widehat{p}_5$ .

It follows that

$$\widehat{p}_1 + \frac{\delta}{1-\delta}\widehat{p}_1 = p_1, \quad \widehat{p}_2 + \frac{\delta}{1-\delta}\widehat{p}_2 = p_2, \quad \widehat{p}_3 + \frac{\delta}{1-\delta}\widehat{p}_3 = p_3, \quad \text{and} \quad \widehat{p}_4 + \frac{\delta}{1-\delta}\widehat{p}_4 = p_4.$$

These imply

$$\widehat{p}_1 = (1-\delta)p_1, \quad \widehat{p}_2 = (1-\delta)p_2, \quad \widehat{p}_3 = (1-\delta)p_3, \quad \text{and} \quad \widehat{p}_4 = (1-\delta)p_4,$$

which in turn imply

$$\widehat{p}_5 = \delta p_1, \quad \widehat{p}_6 = \delta p_2, \quad \widehat{p}_7 = \delta p_3, \quad \text{and} \quad \widehat{p}_8 = \delta p_4.$$

### Proof of Proposition 3

Reverse Bayesianism implies three linearly independent conditions:<sup>43</sup>

$$\begin{aligned} p_2\widehat{p}_1 &= p_1\widehat{p}_2, \\ p_3\widehat{p}_1 &= p_1\widehat{p}_3, \\ \text{and } p_4\widehat{p}_1 &= p_1\widehat{p}_4. \end{aligned}$$

Summing the left- and right-hand sides, and adding  $p_1\widehat{p}_1$  to each side, yields

$$(p_1 + p_2 + p_3 + p_4)\widehat{p}_1 = (\widehat{p}_1 + \cdots + \widehat{p}_4)p_1.$$

By the unit measure axiom on  $S$  and  $\delta \equiv \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9$ , we have  $\widehat{p}_1 = (1-\delta)p_1$ .

Substituting this back into the reverse Bayesian conditions yields

$$\widehat{p}_1 = (1-\delta)p_1, \quad \widehat{p}_2 = (1-\delta)p_2, \quad \widehat{p}_3 = (1-\delta)p_3, \quad \text{and} \quad \widehat{p}_4 = (1-\delta)p_4.$$

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<sup>43</sup>Note again that the six reverse Bayesianism conditions are not linearly independent. In particular, we can derive the last three conditions from the first three.

By act independence and the definitions of  $\alpha$  and  $\beta$ , we have

$$\begin{aligned}
\widehat{p}_5 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5) = \alpha(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5), \\
\widehat{p}_6 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6) = \alpha(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6), \\
\widehat{p}_7 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_7)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9) = \beta(\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_7), \\
\widehat{p}_8 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9) = \beta(\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8), \\
\text{and } \widehat{p}_9 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9) = \alpha\beta.
\end{aligned}$$

These imply

$$\begin{aligned}
\widehat{p}_5 &= \frac{\alpha}{1-\alpha}(\widehat{p}_1 + \widehat{p}_3), & \widehat{p}_6 &= \frac{\alpha}{1-\alpha}(\widehat{p}_2 + \widehat{p}_4), \\
\widehat{p}_7 &= \frac{\beta}{1-\beta}(\widehat{p}_1 + \widehat{p}_2), & \widehat{p}_8 &= \frac{\beta}{1-\beta}(\widehat{p}_3 + \widehat{p}_4), \\
\text{and } \widehat{p}_9 &= \alpha\beta.
\end{aligned}$$

From  $\widehat{p}_9 = \alpha\beta$ , it follows that  $\delta \equiv \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9 = \alpha + \beta - \alpha\beta$ , which implies  $1 - \delta = (1 - \alpha)(1 - \beta)$ . (Observe that  $1 - \delta = (1 - \alpha)(1 - \beta)$  also follows directly from act independence.) It follows that

$$\begin{aligned}
\widehat{p}_1 &= (1 - \alpha)(1 - \beta)p_1, & \widehat{p}_2 &= (1 - \alpha)(1 - \beta)p_2, \\
\widehat{p}_3 &= (1 - \alpha)(1 - \beta)p_3, & \text{and } \widehat{p}_4 &= (1 - \alpha)(1 - \beta)p_4,
\end{aligned}$$

and in turn that

$$\begin{aligned}
\widehat{p}_5 &= \alpha(1 - \beta)(p_1 + p_3), & \widehat{p}_6 &= \alpha(1 - \beta)(\widehat{p}_2 + \widehat{p}_4), \\
\widehat{p}_7 &= \beta(1 - \alpha)(\widehat{p}_1 + \widehat{p}_2), & \widehat{p}_8 &= \beta(1 - \alpha)(\widehat{p}_3 + \widehat{p}_4), \\
\text{and } \widehat{p}_9 &= \alpha\beta.
\end{aligned}$$

## Proof of Corollary 1

We establish in the proof of Proposition 3 that

$$\begin{aligned}\widehat{p}_1 &= (1 - \delta)p_1, & \widehat{p}_2 &= (1 - \delta)p_2, \\ \widehat{p}_3 &= (1 - \delta)p_3, & \text{and } \widehat{p}_4 &= (1 - \delta)p_4.\end{aligned}$$

We also observe that  $1 - \delta = (1 - \alpha)(1 - \beta)$  and  $\delta = \alpha + \beta - \alpha\beta$ . It follows that

$$\begin{aligned}\widehat{p}_5 &= \alpha(1 - \beta)(p_1 + p_3) = \frac{\alpha}{1 - \alpha}(1 - \delta)(p_1 + p_3) = (\delta - \beta)(p_1 + p_3), \\ \widehat{p}_6 &= \alpha(1 - \beta)(\widehat{p}_2 + \widehat{p}_4) = \frac{\alpha}{1 - \alpha}(1 - \delta)(\widehat{p}_2 + \widehat{p}_4) = (\delta - \beta)(\widehat{p}_2 + \widehat{p}_4), \\ \widehat{p}_7 &= \beta(1 - \alpha)(\widehat{p}_1 + \widehat{p}_2) = \frac{\beta}{1 - \beta}(1 - \delta)(\widehat{p}_1 + \widehat{p}_2) = (\delta - \alpha)(\widehat{p}_1 + \widehat{p}_2), \\ \widehat{p}_8 &= \beta(1 - \alpha)(\widehat{p}_3 + \widehat{p}_4) = \frac{\beta}{1 - \beta}(1 - \delta)(\widehat{p}_3 + \widehat{p}_4) = (\delta - \alpha)(\widehat{p}_3 + \widehat{p}_4), \\ \text{and } \widehat{p}_9 &= \alpha + \beta - \delta.\end{aligned}$$

## Proof of Proposition 4

(i) Take any  $s \in S$ . By reverse Bayesianism, we have  $|S| - 1$  linearly independent equations:

$$\widehat{p}(t) = \frac{p(t)}{p(s)}\widehat{p}(s), \quad \forall t \in S, t \neq s. \quad (4.1)$$

By the definition of  $\delta$  and the unit measure axiom on  $\widehat{S}$ , we have

$$\sum_{t \in S} \widehat{p}(t) = 1 - \delta. \quad (4.2)$$

Substituting (4.1) into (4.2), we have

$$\widehat{p}(s) + \sum_{t \in S: t \neq s} \frac{p(t)}{p(s)}\widehat{p}(s) = 1 - \delta,$$

which implies

$$\widehat{p}(s) = \frac{(1 - \delta)p(s)}{\sum_{t \in S} p(t)} = (1 - \delta)p(s), \quad (4.3)$$

where the last equality follows from the unit measure axiom on  $S$ .

(ii) Take any  $s \in \Delta$ . By act independence,

$$\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i)).$$

Observe that  $\widehat{p}(A_l(s^l)) = \widehat{p}(A_l(z_k)) = \delta$  and  $\bigcap_{i \neq l} A_i(s^i) = L(s) \cup \{s\}$ . It follows that

$$\begin{aligned} \widehat{p}(s) &= \delta \prod_{i \neq l} \widehat{p}(A_i(s^i)) = \delta \widehat{p}\left(\bigcap_{i \neq l} A_i(s^i)\right) \\ &= \delta \widehat{p}(L(s) \cup \{s\}) = \delta [\widehat{p}(L(s)) + \widehat{p}(s)], \end{aligned}$$

which implies

$$\widehat{p}(s) = \frac{\delta}{1 - \delta} \widehat{p}(L(s)). \quad (4.4)$$

Observe that  $L(s)$  is the union of all  $t \in S$  such that  $t^i = s^i$  for all  $i \neq l$ . It follows that

$$\widehat{p}(L(s)) = \sum_{t \in L(s)} \widehat{p}(t) = \sum_{t \in L(s)} (1 - \delta)p(t) = (1 - \delta)p(L(s)), \quad (4.5)$$

where the second equality follows from (4.3). Substituting (4.5) back into (4.4), we have

$$\widehat{p}(s) = \delta p(L(s)).$$

## Proof of Proposition 5

(i) Observe that  $\sum_{j=1}^n \widehat{\pi}_{lj} z_j = \sum_{j \neq k} \widehat{\pi}_{lj} z_j + \delta z_k$ . By Proposition 4,

$$\sum_{j \neq k} \widehat{\pi}_{lj} z_j = \sum_{j \neq k} \left[ \sum_{s \in \widehat{S}: s^l = z_j} \widehat{p}(s) \right] z_j = \sum_{j \neq k} \left[ \sum_{s \in S: s^l = z_j} (1 - \delta)p(s) + \sum_{s \in \Delta: s^l = z_j} \delta p(L(s)) \right] z_j.$$

Observe that  $s^l = z_k$  for all  $s \in \Delta$ . It follows that, for all  $j \neq k$ ,

$$\sum_{s \in \Delta: s^l = z_j} \delta p(L(s)) = 0.$$

Thus, we have

$$\sum_{j \neq k} \widehat{\pi}_{lj} z_j = \sum_{j \neq k} \left[ \sum_{s \in S: s^l = z_j} (1 - \delta) p(s) \right] z_j = \sum_{j \neq k} (1 - \delta) \pi_{lj} z_j = \sum_{j=1}^n (1 - \delta) \pi_{lj} z_j,$$

where the last equality follows from  $\pi_{lk} = 0$ . Hence,  $\sum_{j=1}^n \widehat{\pi}_{lj} z_j = \sum_{j=1}^n (1 - \delta) \pi_{lj} z_j + \delta z_k$ .

(ii) Take any  $i \neq l$  and any  $j$ . By Proposition 4,

$$\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s) = \sum_{s \in S: s^i = z_j} (1 - \delta) p(s) + \sum_{s \in \Delta: s^i = z_j} \delta p(L(s)).$$

Observe that  $L(s)$  is the union of all  $t \in S$  such that  $t^i = s^i$  for all  $i \neq l$ . Thus,

$$\sum_{s \in \Delta: s^i = z_j} p(L(s)) = \sum_{t \in S: t^i = z_j} p(t).$$

Hence,

$$\widehat{\pi}_{ij} = \sum_{s \in S: s^i = z_j} (1 - \delta) p(s) + \sum_{s \in S: s^i = z_j} \delta p(s) = \sum_{s \in S: s^i = z_j} p(s) = \pi_{ij}.$$

It follows that  $\widehat{x}_i = \widetilde{x}_i$  for all  $i \neq l$ .

## Proof of Corollary 2

By Proposition 5,  $\xi(\widehat{x}_l) = \sum_{j=1}^n (1 - \delta) \pi_{lj} z_j + \delta z_k$ . Observe that  $\xi(\widetilde{x}_l) = \sum_{j=1}^n \pi_{lj} z_j$ . It follows that  $\xi(\widehat{x}_l) = \xi(\widetilde{x}_l)$  if and only if  $z_k = \sum_{j=1}^n \pi_{lj} z_j$ . Because  $\xi^l(x_i) > 0$  for all  $x_i$ , we have  $\widehat{x}_l = \widetilde{x}_l$  if and only if  $z_k = \sum_{j=1}^n \pi_{lj} z_j$ .

## Proof of Proposition 6

By Proposition 5 and  $\widehat{x}_l = \widehat{\widehat{x}}_l$ , we have  $\xi(\widehat{x}_l) = \sum_{j=1}^n (1 - \delta)\pi_{lj}z_j + \delta z_k$ . It follows that

$$\delta = \frac{\xi(\widehat{x}_l) - \sum_{j=1}^n \pi_{lj}z_j}{z_k - \sum_{j=1}^n \pi_{lj}z_j}.$$

Observe that  $\xi(\widehat{x}_l) = -c'(\widehat{x}_l)/\tau'(\widehat{x}_l)$ . Thus,

$$\delta = \frac{c'(\widehat{x}_l) + \sum_{j=1}^n \pi_{lj}z_j\tau'(\widehat{x}_l)}{\sum_{j=1}^n \pi_{lj}z_j\tau'(\widehat{x}_l) - z_k\tau'(\widehat{x}_l)}.$$

## Proof of Proposition 7

Take any  $s \in S$ . By reverse Bayesianism, we have  $|S| - 1$  linearly independent equations:

$$p(t)\widehat{p}(E(s)) = p(s)\widehat{p}(E(t)), \quad \forall t \in S, t \neq s.$$

Summing the left- and right-hand sides, and adding  $p(s)\widehat{p}(E(s))$  to each side, yields

$$\widehat{p}(E(s)) \sum_{t \in S} p(t) = p(s) \sum_{t \in S} \widehat{p}(E(t)).$$

By the unit measure axioms on  $S$  and  $\widehat{S}$ , we have

$$\widehat{p}(E(s)) = p(s). \tag{7.1}$$

Take any  $s_j \in E(s)$ ,  $j \in \{1, \dots, n\}$ . By act independence,  $\widehat{p}(s_j) = \prod_{i=1}^{m+1} \widehat{p}(A_i(s_j^i))$ . Observe that  $\widehat{p}(A_{m+1}(s_j^{m+1})) = \widehat{p}(A_{m+1}(z_j)) = \delta_j$  and  $\bigcap_{i=1}^m A_i(s_j^i) = E(s)$ . It follows that

$$\widehat{p}(s_j) = \delta_j \prod_{i=1}^m \widehat{p}(A_i(s_j^i)) = \delta_j \widehat{p}\left(\bigcap_{i=1}^m A_i(s_j^i)\right) = \delta_j \widehat{p}(E(s)). \tag{7.2}$$

Substituting (7.1) into (7.2), we have  $\widehat{p}(s_j) = \delta_j p(s)$ .

## Proof of Proposition 8

(i) Recall that  $\{E(s) : s \in S\}$  forms a partition of  $\widehat{S}$ . Take any  $i \neq m+1$  and any  $j$ . By Proposition 7,

$$\begin{aligned}\widehat{\pi}_{ij} &= \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s) = \sum_{s \in \widehat{S}: s^i = z_j} \left[ \sum_{s_l \in E(s)} \widehat{p}(s_l) \right]. \\ &= \sum_{s \in \widehat{S}: s^i = z_j} \left[ \sum_{l=1}^n \delta_l p(s) \right] = \sum_{s \in \widehat{S}: s^i = z_j} p(s) \left[ \sum_{l=1}^n \delta_l \right].\end{aligned}$$

Note that  $\sum_{l=1}^n \delta_l = 1$ . Thus,  $\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} p(s) = \pi_{ij}$ . It follows that  $\widehat{x}_i = \widetilde{x}_i$  for all  $i \neq m+1$ .

(ii) By definition,  $\widehat{\pi}_{m+1,j} = \delta_j$  for all  $j = 1, \dots, n$ . Hence,  $\widehat{x}_{m+1} = \xi^{-1} \left( \sum_{j=1}^n \delta_j z_j \right)$ .

## Proof of Proposition 9

Observe that  $\widehat{h}_{m+1}(x_{m+1}) = \sum_{j=1}^n \widehat{\pi}_{m+1,j} z_j \tau(x_{m+1})$  and  $\widehat{x}_{m+1} = \widehat{\widetilde{x}}_{m+1} = \xi^{-1} \left( \sum_{j=1}^n \widehat{\pi}_{m+1,j} z_j \right)$ . The latter implies  $\xi(\widehat{x}_{m+1}) = \sum_{j=1}^n \widehat{\pi}_{m+1,j} z_j$ . Thus,  $\widehat{h}_{m+1}(x_{m+1}) = \xi(\widehat{x}_{m+1}) \tau(x_{m+1})$ . Recall that  $\xi(x_i) \equiv -c'(x_i)/\tau'(x_i)$ . Hence,  $\widehat{h}_{m+1}(x_{m+1}) = -\frac{c'(\widehat{x}_{m+1})}{\tau'(\widehat{x}_{m+1})} \tau(x_{m+1})$ .

## Proof of Proposition 10

(i) Take any  $s \in S$ . By reverse Bayesianism, we have  $|S| - 1$  linearly independent equations:

$$p(t)\widehat{p}(s) = p(s)\widehat{p}(t), \quad \forall t \in S, t \neq s.$$

Summing the left- and right-hand sides, and adding  $p(s)\widehat{p}(s)$  to each side, yields

$$\widehat{p}(s) \sum_{t \in S} p(t) = p(s) \sum_{t \in S} \widehat{p}(t).$$

Observe that  $\sum_{t \in S} p(t) = 1$  and  $\sum_{t \in S} \widehat{p}(t) = 1 - \delta = \prod_{i=1}^m (1 - \alpha_i)$ . Thus,

$$\widehat{p}(s) = (1 - \delta)p(s) = \left( \prod_{i=1}^m (1 - \alpha_i) \right) p(s). \tag{10.1}$$

(ii) Take any  $s \in \Delta$  such that  $I(s) = \{k\}$  for any  $k \in \{1, \dots, m\}$ . By act independence,  $\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i))$ . Observe that  $\widehat{p}(A_k(s^k)) = \widehat{p}(A_k(z_{n+1})) = \alpha_k$ . Thus,

$$\widehat{p}(s) = \alpha_k \prod_{i \in \bar{I}(s)} \widehat{p}(A_i(s^i)).$$

Observe that  $I(s) = \{k\}$  implies  $\bigcap_{i \in \bar{I}(s)} A_i(s^i) = C(s) \cup \{s\}$ . Hence,

$$\begin{aligned} \widehat{p}(s) &= \alpha_k \prod_{i \in \bar{I}(s)} \widehat{p}(A_i(s^i)) = \alpha_k \widehat{p}\left(\bigcap_{i \in \bar{I}(s)} A_i(s^i)\right) \\ &= \alpha_k \widehat{p}(C(s) \cup \{s\}) = \alpha_k (\widehat{p}(C(s)) + \widehat{p}(s)), \end{aligned}$$

which implies

$$\widehat{p}(s) = \frac{\alpha_k}{1 - \alpha_k} \widehat{p}(C(s)). \quad (10.2)$$

Observe that  $C(s)$  is the union of all  $t \in S$  such that  $t^i = s^i$  for all  $i \in \bar{I}(s)$ . It follows that

$$\widehat{p}(C(s)) = \sum_{t \in C(s)} \widehat{p}(t) = \sum_{t \in C(s)} (1 - \delta)p(t) = (1 - \delta)p(C(s)), \quad (10.3)$$

where the second equality follows from (10.1). Substituting (10.3) back into (10.2), we have

$$\widehat{p}(s) = \frac{\alpha_k}{1 - \alpha_k} (1 - \delta)p(C(s)) = \alpha_k \prod_{i \in \bar{I}(s)} (1 - \alpha_i)p(C(s)),$$

where the last equality follows from  $1 - \delta = \prod_{i=1}^m (1 - \alpha_i)$ .

Next take any  $s \in \Delta$  such that  $I(s) = \{k, l\}$  for any  $\{k, l\} \subset \{1, \dots, m\}$ . By act independence,  $\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i))$ . Observe that  $\widehat{p}(A_k(s^k)) = \widehat{p}(A_k(z_{n+1})) = \alpha_k$ . Thus,

$$\widehat{p}(s) = \alpha_k \prod_{i \in \{\bar{I}(s) \cup \{l\}\}} \widehat{p}(A_i(s^i)).$$

Observe that  $I(s) = \{k, l\}$  implies  $\bigcap_{i \in \{\bar{I}(s) \cup \{l\}\}} A_i(s^i) = D(s) \cup \{s\}$ , where  $D(s) \equiv \{r \in \Delta : r^i = s^i, \forall i \in \{\bar{I}(s) \cup \{l\}\}\}$ . Hence,

$$\begin{aligned}
\widehat{p}(s) &= \alpha_k \prod_{i \in \{\bar{I}(s) \cup \{l\}\}} \widehat{p}(A_i(s^i)) = \alpha_k \widehat{p}\left(\bigcap_{i \in \{\bar{I}(s) \cup \{l\}\}} A_i(s^i)\right) \\
&= \alpha_k \widehat{p}(D(s) \cup \{s\}) = \alpha_k (\widehat{p}(D(s)) + \widehat{p}(s)),
\end{aligned}$$

which implies

$$\widehat{p}(s) = \frac{\alpha_k}{1 - \alpha_k} \widehat{p}(D(s)). \quad (10.4)$$

Observe further that  $I(r) = \{l\}$  for all  $r \in D(s)$ . It follows that

$$\begin{aligned}
\widehat{p}(D(s)) &= \sum_{t \in D(s)} \widehat{p}(t) \\
&= \sum_{t \in D(s)} \frac{\alpha_l}{1 - \alpha_l} (1 - \delta) p(C(t)) \\
&= \frac{\alpha_l}{1 - \alpha_l} (1 - \delta) p(C(s)).
\end{aligned} \quad (10.5)$$

Substituting (10.5) back into (10.4), we have

$$\begin{aligned}
\widehat{p}(s) &= \frac{\alpha_k}{1 - \alpha_k} \frac{\alpha_l}{1 - \alpha_l} (1 - \delta) p(C(s)). \\
&= \alpha_k \alpha_l \prod_{i \in \bar{I}(s)} (1 - \alpha_i) p(C(s)).
\end{aligned}$$

Proceeding in this fashion to consider  $s \in \Delta$  such that  $I(s)$  is an  $\iota$ -element subset of  $\{1, \dots, m\}$  for all  $\iota = 3, \dots, m - 1$ , we establish that

$$\widehat{p}(s) = \left( \prod_{i \in I(s)} \alpha_i \right) \left( \prod_{i \in \bar{I}(s)} (1 - \alpha_i) \right) p(C(s))$$

for all  $s \in \Delta$  such that  $I(s) \subset \{1, \dots, m\}$ .

(iii) Take the  $s \in \Delta$  such that  $I(s) = \{1, \dots, m\}$ . By act independence,  $\widehat{p}(s) = \prod_{i=1}^m \widehat{p}(A_i(s^i))$ . Observe that  $\widehat{p}(A_i(s^i)) = \widehat{p}(A_i(z_{n+1})) = \alpha_i$  for all  $i \in I(s)$ . Because  $I(s) = \{1, \dots, m\}$ , we have  $\widehat{p}(s) = \prod_{i=1}^m \alpha_i$ .

## Proof of Proposition 11

Take any  $i \in \{1, \dots, m\}$ . Observe that

$$\widehat{x}_i = \xi^{-1} \left( \sum_{j=1}^{n+1} \widehat{\pi}_{ij} z_j \right) = \xi^{-1} \left( \sum_{j=1}^n \widehat{\pi}_{ij} z_j + \alpha_i z_{n+1} \right). \quad (11.1)$$

Let  $\Gamma(\alpha_l, s) \equiv \left( \prod_{l \in I(s)} \alpha_l \right) \left( \prod_{l \in \bar{I}(s)} (1 - \alpha_l) \right)$  for all  $s \in \Delta$ . By Proposition 10,

$$\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s) = \sum_{s \in S: s^i = z_j} \prod_{l=1}^m (1 - \alpha_l) p(s) + \sum_{s \in \Delta: s^i = z_j} \Gamma(\alpha_l, s) p(C(s)),$$

for all  $j \neq n + 1$ . Observe that

$$\sum_{s \in S: s^i = z_j} \prod_{l=1}^m (1 - \alpha_l) p(s) = \prod_{l=1}^m (1 - \alpha_l) \sum_{s \in S: s^i = z_j} p(s) = (1 - \delta) \pi_{ij}$$

and that

$$\begin{aligned} \sum_{s \in \Delta: s^i = z_j} \Gamma(\alpha_l, s) p(C(s)) &= \sum_{s \in \Delta: s^i = z_j} \left( \prod_{l \in I(s)} \alpha_l \right) \left( \prod_{l \in \bar{I}(s)} (1 - \alpha_l) \right) p(C(s)) \\ &= \sum_{s \in \Delta: s^i = z_j} \frac{\prod_{l \in I(s)} \alpha_l}{\prod_{l \in I(s)} (1 - \alpha_l)} (1 - \delta) p(C(s)) \\ &= \sum_{I \subset \{1, \dots, m\} \setminus \{i\}} \frac{\prod_{l \in I} \alpha_l}{\prod_{l \in I} (1 - \alpha_l)} (1 - \delta) \pi_{ij} \\ &= \frac{1 - \prod_{l \neq i} (1 - \alpha_l)}{\prod_{l \neq i} (1 - \alpha_l)} (1 - \delta) \pi_{ij}. \end{aligned}$$

Thus,

$$\begin{aligned} \widehat{\pi}_{ij} &= (1 - \delta) \pi_{ij} + \frac{1 - \prod_{l \neq i} (1 - \alpha_l)}{\prod_{l \neq i} (1 - \alpha_l)} (1 - \delta) \pi_{ij} = (1 - \delta) \pi_{ij} \left( 1 + \frac{1 - \prod_{l \neq i} (1 - \alpha_l)}{\prod_{l \neq i} (1 - \alpha_l)} \right) \\ &= (1 - \delta) \pi_{ij} \left( \frac{1}{\prod_{l \neq i} (1 - \alpha_l)} \right) = (1 - \delta) \pi_{ij} \left( \frac{1 - \alpha_i}{1 - \delta} \right) = (1 - \alpha_i) \pi_{ij}. \end{aligned} \quad (11.2)$$

Substituting (11.2) back into (11.1), we have  $\widehat{x}_i = \xi^{-1} \left( \sum_{j=1}^n (1 - \alpha_i) \pi_{ij} z_j + \alpha_i z_{n+1} \right)$ .

### Proof of Corollary 3

By Proposition 11,  $\xi(\widehat{x}_i) = \sum_{j=1}^n (1-\alpha_i)\pi_{ij}z_j + (1-\alpha_i)z_{n+1}$ . Observe that  $\xi(\widetilde{x}_i) = \sum_{j=1}^n \pi_{ij}z_j$ . It follows that  $\xi(\widehat{x}_i) = \xi(\widetilde{x}_i)$  if and only if  $z_{n+1} = \sum_{j=1}^n \pi_{ij}z_j$ . Because  $\xi'(x_i) > 0$  for all  $x_i$ , we have  $\widehat{x}_i = \widetilde{x}_i$  if and only if  $z_{n+1} = \sum_{j=1}^n \pi_{ij}z_j$ .

### Proof of Proposition 12

Take any  $i \in \{1, \dots, m\}$ . By Proposition 11,  $\widehat{x}_i = \widetilde{x}_i = \xi^{-1}\left(\sum_{j=1}^n (1-\alpha_i)\pi_{ij}z_j + \alpha_i z_{n+1}\right)$ , which implies  $\xi(\widehat{x}_i) = \sum_{j=1}^n (1-\alpha_i)\pi_{ij}z_j + \alpha_i z_{n+1}$ . It follows that

$$\alpha_i = \frac{\xi(\widehat{x}_i) - \sum_{j=1}^n \pi_{ij}z_j}{z_{n+1} - \sum_{j=1}^n \pi_{ij}z_j}.$$

Observe that  $\xi(\widehat{x}_i) = -c'(\widehat{x}_i)/\tau'(\widehat{x}_i)$ . Thus,

$$\alpha_i = \frac{c'(\widehat{x}_i) + \sum_{j=1}^n \pi_{ij}z_j\tau'(\widehat{x}_i)}{\sum_{j=1}^n \pi_{ij}z_j\tau'(\widehat{x}_i) - z_{n+1}\tau'(\widehat{x}_i)}.$$

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