Why is Debt Debt?

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Abstract

JEL Classification:
1. Introduction

The core insight of the Modigliani-Miller propositions\(^1\) is that in a world of no taxes, transactions costs, or information asymmetries, the manner in which the firm is financed, the debt/equity choice, has no effect on the value of the firm. Since those papers numerous authors have shown that when these core assumptions are relaxed there may be an optimal – in the sense of value maximizing– mix of debt and equity. One strand in this literature has been to point out that Debt can be a mechanism that disciplines managers by raising the cost of pursuing non-value maximizing operating policies. All of these authors have assumed, correctly, that debt, relative to equity, enjoys a prior claim on the firm’s assets. What has not heretofore been addressed is how these claims are adjudicated when the payment is not made.

Finance commonly treats both stockholders and bondholders as "investors" in the firm. An entire literature is devoted to valuing debt and equity claims in which their legal status is ignored, all that matters is that debt has a first claim on the firm upon liquidation. Law, however, draws a fundamental distinction between people who provide capital by purchasing stock and people who provide capital by purchasing equity. To all intents and purposes, people who invest by buying stock have a claim on firm’s directors only if a court determines that they have violated a fiduciary duty. If they are found to have behaved in a manner consistent with a set of standards, the court will not even try to "second guess" the managers. This is the Business Judgement Rule. On the other hand, people who invest by purchasing bonds can control managerial behavior by a set of "covenants" or contractual terms. These restrictions are determined when the bond is issued. In contrast to the restrictions imposed by the duties to stockholders, which require an ex-post evaluation of the action before liability is determined, the ex-ante restrictions imposed by the bond covenant are more easily enforced.

\(^1\)Cite to M-M 1958
This paper addresses the role that is played by the ex-ante nature of the bond indenture. One can easily imagine a legal system in which bondholders, in terms of protection from bad acts by managers, are treated the same as equity investors. Either both have the protections provided by fiduciary duties, or both have the protections provided by covenants.\(^2\) That is not the system that we have. Just as there as the board has few, if any, fiduciary duties to the bondholders, it is hard to imagine a contract between the outside shareholders and the board that, for example, required the board to build a warehouse in Austin no matter what other alternative uses for the funds might arise. This situation is captured by the old saw that the firm is "republic, not a democracy."

Finally, we are able to elucidate a connection between the Business Judgement Rule (BJR) and the legal regime that governs obligations. In brief, to the extent that it is optimal to have a BJR, it will be optimal to have a strong mechanism for entrepreneurs to commit to good behavior. Giving contractual protection to debt may serve that purpose.

This distinction between the way shareholders and debtholders are protected is an example of the common rules vs. standards distinction. Law provides for two different mechanisms for enforcing claims. In the case of debt and equity, perhaps for historic reasons\(^3\), these two different legal standards are applied in the case of debt and equity. For the most part, debt holders claims are evaluated in the "rules" regime, while the claims of equity holders are heard in the "standard of behavior" regime (fiduciary duties)\(^4\).

\(^2\) We want to make clear that we are distinguishing between the definition of the payoffs resulting from the firm’s cash flows, and any additional payments that might accrue to investors if a managerial act is "bad" for investors.

\(^3\) Under common law there are two types of court. Courts of Law can assess only monetary damages, and cases are commonly heard with a jury. Contracts, and for these purposes a bond is a contract, were heard in Law courts. Courts of Equity, on the other hand, were free to craft many kinds of non-monetary damages, and cases in these courts were heard before a judge without a jury. Disputes between partners, or shareholders, were heard in these courts. While in most cases Equity and Law courts were merged in the 19th century, the system still does have some force. The most famous example of this is, of course, the Delaware Court of Chancery, where many corporate law cases in the US are heard, and in which there is no jury.

\(^4\) This distinction was pointed out by, among others Oliver Williamson — insert cite to williamson if proceedings paper
2. The Basic Model

An entrepreneur with assets $A > 0$ wishes to raise funds for an investment project. Funds will be raised with possible debt and equity and the project requires an investment outlay of $C$ dollars. The cash flow technology has three stages and two possible returns, $R \in \{H, 0\}$, where $H > C$. In the first stage the entrepreneur begins to set up the project and gathers funds from outside investors. Funds are raised predicated on a base project which generates the return $H$ with probability $\theta$. When this stage is complete the entrepreneur is given one of two options or paths through observing a private signal in stage 2. On path 1, he learns that the high return will occur with probability $(\theta + x_1)$ and that the low return will occur with probability $(1 - \theta - x_1)$. On path 2, he learns that the high return will occur with probability $(\theta - x_2)$ and that the low return will occur with probability $(1 - \theta + x_2)$. He observes path 1 with probability $\gamma$ and path 2 with probability $(1 - \gamma)$. On path 2, the decision maker receives a private benefit $bx_2$, $b > 0$. This private benefit will be thought of as being non-monetary, non-verifiable and nontransferable. All parties observe whether $x_i \neq 0$, but the value of $x_i$ is private information to the decision maker. The cost of action $x_i$ to the entrepreneur is given by $c_i x_i^2 / 2$. If path 1 is observed, then, from the viewpoint of the firm, it is best for the entrepreneur to select $x > 0$. However, if path 2 is observed, then, under our assumptions, it is best for the firm to have $x_2 = 0$.

Returns accrue in stage 3. All litigation activities take place in a stage 3 after returns have been observed. All parties know the technology and the probability distributions.

Funds $C$ consist of debt, $D$, and equity, $E$, where $C = D + E$. We denote the face value of debt as $F$ and the fraction of returns contributed by outside equity holders is denoted as $s$. In what follows, we assume that all litigation costs are zero, and that if a debt or an equity suit is found in favor of the plaintiff, there is an exogenous probability $p \in (0, 1]$ that the plaintiff will retrieve all funds from the judgement. We also assume that the entrepreneur has sufficient assets to pay off
any debt or equity suit.

2.1. Triggers for Suits

A necessary condition for a debt or equity holder to file a suit is that a low return is observed. If a such an agent has protection by a covenant standard, then we assume that the agent writes a stipulation into the equity or debt contract stating that the entrepreneur will not take a private action, defined by $x_i > 0$. If the covenant is broken the entrepreneur is assumed to pledge his assets $A$ as a personal bond against default. Under our assumptions on information, $x_i > 0$ is detected with probability one. Under a covenant standard, a suit will be triggered by a low return and $x_i > 0$ regardless of whether the entrepreneur has observed path 1 or path 2.

A debt or equity holder covered by a fiduciary duty standard may also sue the entrepreneur if she feels that he has not carried out his fiduciary duty. A lack of fiduciary duty is a reality if the entrepreneur observes path $2$ and sets $x_2 > 0$. Otherwise, there is no true violation. However, if a debt or equity holder covered by a fiduciary duty standard observes a low return and that $x_i \neq 0$, they will sue. If a suit is brought to the court, we let $\alpha_2$ denote the probability that the court will correctly assess that path $2$ has been observed, conditional on the entrepreneur observing path two. If a suit is brought to the court and the entrepreneur observed path one, the probability that the court will incorrectly assess that path two has been observed is denoted as $\alpha_1$.

3. Solutions to the Stage 1, Stage 3 and Stage 2 Problems

3.1. Stage 3

In stage 3, the court must assess damages if a suit is brought forward. First consider the case of a covenant standard. In this case, all that is required is that $x_i > 0$. This is observed with perfection, but damages are paid to debt or equity holders with probability $p_i$ if $x_i > 0$ and a low return is
observed. If the plaintiff is a debt holder the firm will face expected damages of

\[ L_d^c = pF. \]

On the other hand if the plaintiff is an equity holder expected damage is the difference between expected returns from the base project and realized returns, where realized returns would be zero. Thus, expected damage for either path is

\[ L_e^c = ps\theta(H - F). \]

If the suit is a fiduciary duty suit, conditional on a low return being observed and \( x_1 > 0 \) the court must assess damages if it believes that the entrepreneur has violated his fiduciary duty. Consider the case of an equity holder. In this case, the court must believe that the entrepreneur has observed path 2 and has detracted from the probability of a high return in the amount of \( x_2 \). True expected damages are then given by the \( \max\{sx_2(H - F), sx_2^\theta\} \). Let \( x_2^* \) be the court's assessment of the entrepreneur's private action variable, so that the entrepreneur would face damages based on \( M^* = \max\{sx_2^*(H - F), sx_2^\theta\} \). However, because \( bx_2^* \) is non-verifiable, \( M^* = sx_2^*(H - F) \). The estimate \( x_2^* \) defines the court's standard for damages. It represents the \( x_2 \) that would be chosen in path 2, if the entrepreneur faced a fixed \( M^* \) to be used in stage 3. If the entrepreneur actually observed path 2, a low return was observed, and \( x_2 > 0 \), then the court correctly sees that path 2 has been observed with probability \( \alpha_2 \). Expected damages from a FD suit would be

\[ L_e^F = ps\alpha_2x_2^*(H - F). \]
If the same conditions are met except that path 1 was observed, the court incorrectly assesses the path with probability $\alpha_1$ and expected damages from a fiduciary duty suit are

$$L_{ei}^{F} = ps\alpha_1 x_2^{*}(H - F).$$

In the case of a debt holder under fiduciary duty, expected damages to the firm observing path $i$ are

$$L_{di}^{F} = \alpha_i p F, \text{ for } i = 1, 2.$$

We assume that $\alpha_2 > \alpha_2$.

There are four possible legal arrangements that would be of theoretical interest. Let $FD$ denote a fiduciary duty standard and let $Cov$ denote a covenant standard. Our current institutional arrangement is for debt to have a covenant standard and for equity to have a fiduciary duty standard. We can write this as $(Cov, FD)$ and term this the base model, where the first entry refers to the standard for debt and the second refers to that for equity. The other possible arrangements are $(Cov, Cov), (FD, FD)$, and $(FD, Cov)$. In all of these cases, debt is senior to equity. In what follows let us use the notation $L_{di}$ to denote the expected damages to the firm under a debt suit and $L_{ei}$ denotes the same under an equity suit, conditional on the firm observing path $i$. If, for example, the legal arrangement is $(Cov, FD)$, then the stage 3 expected damages to the firm are $L_{di} = L_{d}^{*} = pF$, $i = 1, 2$, for debt and $L_{ei}^{F} = \alpha_i x_2^{*}(H - F)$, $i = 1, 2$, for equity.
3.2. Stage 2

Next, consider stage 2. Assuming that path one has been observed, the entrepreneur will choose \( x_1 \) as the solution to

\[
\max_{\{x_1\}} (1 - s)(H - F)(\theta + x_1) - cx_1^2/2 - (1 - \theta - x_1)(L_{e1} + L_{d1}).
\]

The first order condition for this problem is

\[-cx_1 + [(1 - s)(H - F) + L_{e1} + L_{d1}] \leq 0,\]

where equality holds if \( x_1 > 0 \). At an interior solution,

\[
x_1 = c_1^{-1}[(1 - s)(H - F) + L_{e1} + L_{d1}]. \tag{1}
\]

Next, consider the case where path 2 is observed by the entrepreneur. The choice of \( x_2 \) is defined by

\[
\max_{\{x_2\}} (1 - s)(H - F)(\theta - x_2) + bx_2 - (1 - \theta + x_2)(L_{e2} + L_{d2})
\]

and the corresponding (FOC)

\[b - cx_2 - [(1 - s)(H - F) + L_{e2} + L_{d2}] \leq 0,\]

where equality holds if \( x_2 > 0 \). At an interior solution,

\[
x_2 = c_2^{-1}[b - (1 - s)(H - F) + L_{e2} + L_{d2}]. \tag{2}
\]
3.3. Stage 1

The stage 1 problem begins with the gathering of funds. The expressions for equity and debt can now be expressed. Equity is valued as follows

\[
E = s\gamma(\theta + x_1)(H - F) + \gamma(1 - \theta - x_1)L_{e1} + \\
(1 - \gamma)(\theta - x_2)(H - F) + (1 - \gamma)(1 - \theta + x_2)L_{e2}. 
\]  

(3)

The value of debt is expressed as

\[
D = \gamma(\theta + x_1)F + \gamma(1 - \theta - x_1)L_{d1} + (1 - \gamma)(\theta - x_2)F + (1 - \gamma)(1 - \theta + x_2)L_{d2}. 
\]  

(4)

Define

\[
g(s, F) = E + D.
\]

Given \((s, F)\), the expressions (3) and (4) uniquely determine \(D\). Thus, \(D\) can be eliminated as a choice variable in the entrepreneur’s stage 1 problem. The entrepreneur’s problem for the case where his assets \(A\) are sufficient to cover any legal damages is then given by

\[
\begin{align*}
\max_{\{s,F\}} (1 - s)\gamma(\theta + x_1)(H - F) - \gamma(1 - \theta - x_1)[L_{e1} + L_{d1}] \\
+ (1 - s)(1 - \gamma)(\theta - x_2)(H - F) - (1 - \gamma)(1 - \theta + x_2)[L_{e2} + L_{d2}] - \sum_{i=1}^{2} c_i x_i + b x_2.
\end{align*}
\]  

(F)

Subject to \(g(s, F) - C = 0\), from (3) – (4).
4. A Benchmark

The first best benchmark can be defined as the solution to the total surplus problem under the assumption that there is no need to finance the project. In this way there are no imperfections entering from the financial market or the legal system. The decision maker would begin in stage one by setting up the project and in stage two would receive information. Given information on the path, the planner would, in path one, solve

$$\max_{x_1} (\theta + x_1)H + (1 - \theta - x_1)(0) - c_1x_1^2/2,$$

so that

$$x_1^o = H/c_1.$$

For path two, we have that the planner solves

$$\max_{x_2} (\theta - x_2)H + (1 - \theta + \eta(x_2))(0) - c_2x_2^2/2,$$

so that

$$-H - c_2x_2 \leq 0$$ with equality if $$x_2^o > 0$$ and we have $$x_2^o = 0.$$

The first best level of expected profit from a time zero viewpoint is then given by

$$\gamma(\theta + x_1^2)H + (1 - \gamma)(\theta - 0)H - c_1x_1^2/2 = \gamma(\theta + x_2^2)H + (1 - \gamma)\theta H - c_1x_1^2/2.$$
5. Analysis of the Base Case

Here we study the allocation produced by the base legal regime of \((Cov, FD)\). I will include parameters which allow us to move to \((Cov, Cov)\) and \((FD, FD)\). In this case, the stage 2 problems specialize as in the following. If path 1 is observed, we have that

\[
x_{1}^{CF} = c_{1}^{-1}[(1 - s)(H - F) + kps\alpha_{1}x_{2}^{*}(H - F) + hpF],
\]

and

\[
\frac{\partial x_{1}^{CF}}{\partial s} = c_{1}^{-1}(H - F)[-1 + kps\alpha_{1}x_{2}^{*}] < 0.\]
\[
\frac{\partial x_{1}^{CF}}{\partial F} = c_{1}^{-1}[-(1 - p) + s(1 - kps\alpha_{1}x_{1}^{*})].
\]

If path 2 is observed, we have that

\[
x_{2}^{CF} = c_{2}^{-1}[b - (H - F)(1 - s) - p(k\alpha_{2}x_{2}^{*}(H - F) + hF)].
\]

We have that

\[
\frac{\partial x_{2}^{CF}}{\partial s} = c_{2}^{-1}(H - F)[1 - kps\alpha_{2}x_{2}^{*}] > 0, \text{ and}
\]
\[
\frac{\partial x_{2}^{CF}}{\partial F} = c_{2}^{-1}[1 - hp - s(1 - p\alpha_{2}x_{2}^{*})].
\]
The financing constraint specializes to

\[
\begin{align*}
\gamma(\theta + x_1)F &+ \gamma(1 - \theta - x_1)hpF + (1 - \gamma)(\theta - x_2)F + (1 - \gamma)(1 - \theta + x_2)hpF \\
+ s\gamma(\theta + x_1)(H - F) &+ \gamma(1 - \theta - x_1)kps\alpha_1 x_2^s(H - F) + s(1 - \gamma)(\theta - x_2)(H - F) \\
+ (1 - \gamma)(1 - \theta + x_2)kps\alpha_2 x_2^s(H - F) - C &= g(\cdot) - C = 0.
\end{align*}
\]

Using the funding constraint, we can write the manager's stage 1 problem as

\[
\max_{\{s,F,\lambda\}} H[\gamma(\theta + x_1) + (1 - \gamma)(\theta - x_2)] - g(s, F) + bx_2 - \sum_{i=1}^{2} c_i x_i + \lambda[g(s, F) - C].
\]

the FOC are

\[
\lambda : g - C = 0
\]

\[
s : H[\gamma \frac{\partial x_1}{\partial s} - (1 - \gamma) \frac{\partial x_2}{\partial s}] - c_1 x_1 \frac{\partial x_1}{\partial x_2} - c_2 x_2 \frac{\partial x_2}{\partial x_2} + b \frac{\partial x_2}{\partial s} - g_s(1 - \lambda) = 0.
\]

\[
F : H[\gamma \frac{\partial x_1}{\partial F} - (1 - \gamma) \frac{\partial x_2}{\partial F}] - c_1 x_1 \frac{\partial x_1}{\partial x_2} - c_2 x_2 \frac{\partial x_2}{\partial x_2} + b \frac{\partial x_2}{\partial F} - g_F(1 - \lambda) = 0.
\]

As we increase \( k \) we move to \((Cov, Cov)\) and as we lower \( h \) we move to \((FD, FD)\). Comparative statics of these parameters tell us about how financing is affected by the legal regime.

6. The discrete Case

We may gain more insight by switching to a discrete model. In this case we give up the continuity that allows us to use calculus, but we still be able to generate some numerical results by making specific assumptions about parameter values. We will see that for some parameter values \((Cov, FD)\) results in better operating decisions, and hence more value for outside investors, but this is not universally the case.
In the discrete model $x_1$ and $x_2$ are not choice variables. The manager is offered an alternative project and if she takes it she give up all of the base project. Alternative 1 is still better than the base project in that the high return will occur with $(\Theta + \delta)$, and project two is worse because the base project because the probability of the high outcome is $(\Theta - \delta)$. Each alternative project has a private benefit of $B$ associated with it. The manager is also endowed with a fixed sum, $X$ which is not invested in the project but is available to pay damages in any litigation. The manager chooses not invest this amount in the firm for exogenous reasons.

In the following we simplify matters by assuming that the cash flow from operations, the payoff, in the Low outcome state is 0. This does not change any intuition, but is does simplify the math somewhat.

6.1. Litigation under FD and Cov

6.1.1. Likelihood of Liability under FD and Cov

**Likelihood of Liability Under FD** In the discrete model the judge is presumed to be a Bayesian. She knows with certainty that there has been a substitution, but does not know whether the substitution was into the good project or the bad project. Since, as we shall see later, actual damages if will only be positive if the Low payout state occurs$^5$, the judge will determine the probability that the manager chose the bad alternative project rather than the good alternative project. This is a function of (1) the likelihood of the low payout from both projects and (2) the probability of getting offered project 2.

The judge knows that if the manager took the good project the probability of getting $L$ is $(1 - (\Theta + \delta))$ and if the manager took the bad project the probability of getting the bad outcome is $(1 - (\Theta - \delta))$. The judge must compute $P(2|L)$. Using Bayes Theorem, the judge can solve for

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$^5$Actually, in order to ensure this, we have to introduce the restriction that $F \leq H$.  

13
$$P(2|L) = \frac{P(L|2)P(2)}{P(L|2)P(2) + P(L|1)P(1)} = \frac{(1 - (\Theta - \delta))(1 - \gamma)}{(1 - (\Theta - \delta))(1 - \gamma) + (1 - (\Theta + \delta))\gamma}$$

(6.1)

Absent any other factor, we assume that knowing the manager did substitute, and observing that the low outcome occurred, the judge uses a random device to find liability $P(2|L)$ percent of the time. Also note that while in principle we should also be concerned with $p(2|H)$, the probability that the judge infers a bad act even though the High cash flow occurred, our case we need not worry this because we will be restricted to cases where the damages if the cash flow is H are 0.

**Business Judgement Rule**  The BJR provides a rebuttable proposition that the board has not violated its fiduciary duties. We capture this in a crude way by viewing the BJR as a ‘thumb on the scale’ toward not finding liability. Thus we assume that before finding liability the judge multiplies $P(2|L)$ by some constant $k$ that is between 0 and 1. Clearly if $k=0$ then there is no liability (the presumption is not rebuttable) and if $k=1$ there is no presumption. we then end up with;

$$P(\text{Liability}|\text{Low}) = k\left(\frac{(1 - (\Theta - \delta))(1 - \gamma)}{(1 - (\Theta - \delta))(1 - \gamma) + (1 - (\Theta + \delta))\gamma}\right)$$

Thus, for example, if $k = .5$, $\Theta = .5$, $\delta = .45$ and $\gamma = .5$, we have:

$$P(\text{Liability}|\text{Low}) = k\left(\frac{(1 - (\Theta - \delta))(1 - \gamma)}{(1 - (\Theta - \delta))(1 - \gamma) + (1 - (\Theta + \delta))\gamma}\right) = 0.475$$

(6.2)

**Likelihood of Liability Under Cov**  If the investors are protected by covenants the likelihood of liability is 1. The court can detect, without error, that the covenant was violated so that liability attaches.
6.1.2. Damages

**Damages to Senior Claims (Bonds)** The senior claim has a promised payment of $F < H$. If the High outcome occurs there are sufficient funds from operations to pay off the debt and the bondholder will not collect any damages. If the Low outcome occurs, then by our simplifying assumption that cash flow in the Low state is 0, the debtholder is entitled to collect damages of $F$. In the discrete model we assume that the manager has $X$ available to pay any damages. Thus, the amount he will collect is $\min(F, X)$. For our first numerical example we assume that $X = 70$.\(^6\)

The effect of legal regime is simply to affect the probability, given that the low outcome has occurred, that the court will assess damages. We assume, for our numerical example, that $X = 70$.

The expected damages, given that the low outcome has been realized, are:

\[
\begin{align*}
\text{ExpDam FD Low} &= k \left( \frac{(1 - (\Theta - \delta))(1 - \gamma)}{(1 - (\Theta - \delta))(1 - \gamma) + (1 - (\Theta + \delta))\gamma} \right) \min(X, F) \\
&= 0.475 \min(X, F) \\
&= 0.475 \min(70, F) \\
\text{ExpDam Cov Low} &= \min(X, F) = \min(70, F)
\end{align*}
\]

(6.3) \hspace{1cm} (6.4) \hspace{1cm} (6.5)

As a preview of our analysis, it is clear that the expected damages paid to the senior claim, will be greater if the senior claim is protected by Cov rather than FD, as long as $X > 0$.

**Damages to the Junior Claim (Equity)** The junior claim does not have well a well defined promised payoff that can be used as the basis for computing damages. Rather, we assume that

\(^6\)Another way to think about this point is to imagine that this is really a story about choice between risky debt and equity. The low payoff is $X$, the high payoff is $H + X$ and the manager has purchased a risk-free senior debt issue that pays $X$ at the end of the period.
the court uses an expectations measure of damages. Assume that the manager had taken the base project. In that case for any value of s and F, the expected payoff to outside equity was:

\[
x\text{stokincb}(s, F) = \begin{pmatrix} 1 - \Theta & \Theta \\ \Theta & 1 - \Theta \end{pmatrix} \begin{pmatrix} 0 \\ s(H - F) \end{pmatrix} = \Theta s(H - F)
\]

(6.6)

In addition to assuming that L=0, we further assume that H=100, which makes 6.6 become;

\[
x\text{stokincb}(s, F) = \begin{pmatrix} .5 & .5 \\ .5 & .5 \end{pmatrix} \begin{pmatrix} 0 \\ s(100 - F) \end{pmatrix} = 0.5s(100 - F)
\]

(6.7)

Clearly if the High payoff occurred the outside shareholders received \( s \max(0, 100 - F) \) and thus the damages are 0. If the low payoff occurred, for these parameter values, the liability is \( 0.5s \max(0, 100 - F) \). How much will collected? It might appear that the amount recovered depends on whether the debt is protected by FD or Cov, but that is not the case. Clearly if the bonds are protected by covenants, before the stock has recovered any damages the bonds will have recovered their damages. But the same is true if the bonds are protected by FD. In that case both the junior and the senior claim has FD protection. However, as in payout from operations so in payout from damages, the senior claim collects its damages first. Thus, any time that there are damages paid to the outside shareholders, damages will have already been paid to the bondholders.

Thus:

\[
\text{ExpDam FD Low} = k \left( \frac{(1 - (\Theta - \delta))(1 - \gamma)}{(1 - (\Theta - \delta))(1 - \gamma) + (1 - (\Theta + \delta))\gamma} \right) \min(X - \min(X, F), \Theta s(H - F))
\]

\[
= 0.475 \min(0.5s(100 - F), (70 - \min(70, F))
\]

(6.8)

\[
\text{ExpDam Cov Low} = 0.475 \min(0.5s(100 - F), (70 - \min(70, F))
\]

(6.9)
Total Damages In our example, then, the total damages, then, under Cov, FD, if the low outcome occurs the expected total damages paid to both stockholders and bondholders will be:

\[
0.475 \left( \min\left(0.5s(100 - F), (70 - \min(70, F))\right) \right) + \min(70, F) \quad (6.10)
\]

Under FD, if the low outcome occurs, the expected total damages will be:

\[
0.475 \left( \min\left(0.5s(100 - F), (70 - \min(70, F))\right) \right) + 0.475 \min(70, F) \quad (6.11)
\]

\[
= 0.475 \left( \min\left(0.5s(100 - F), (70 - \min(70, F))\right) \right) + \min(70, F) \quad (6.12)
\]

6.2. Managers Choice of Project Under FD and Cov

The manager has to choose whether to stick with the base project or switch to the alternative project under the liability regime that he faces. She will choose whichever outcome leads to the highest expected income. Remember, that if she stays with the base project there will be no damages paid, and she will be able to keep all of the money at risk for paying damages. If she stays with the base project, for any given values of \(s\) and \(F\), her expected income is:

\[
\begin{pmatrix}
1 - \Theta & \Theta \\
\end{pmatrix}
\begin{pmatrix}
0 \\
(1 - s)(H - F) \\
\end{pmatrix}
+ X 
\quad (6.13)
\]

\[
= \Theta(1 - s)(H - F) + X 
\quad (6.14)
\]

\[
= 0.5 (H - F)(1 - s) + 70 
\quad (6.15)
\]

6.2.1. Choice if Offered Project 1, FD protection for Senior Claim

If the manager is offered project 1, her expected income from operations (not including the retention of the collateral, nor her private benefit, is given by:
\[
\begin{pmatrix}
1 - (\Theta + \delta) & (\Theta + \delta)
\end{pmatrix}
\begin{pmatrix}
0 \\
(1 - s)(H - F)
\end{pmatrix} = 0.95(H - F)(1 - s)
\]

(6.16)

(6.17)

The incremental expected income from operations is thus

\[
\delta(H - F')(1 - s)
\]

(6.18)

\[
= 0.45(H - F')(1 - s)
\]

(6.19)

Her expected total damages are the expected damages conditional on the low payout times the probability of getting the low payout from project 1:

\[
(\Theta - \delta) 0.475 \left( \min (0.5s(100 - F), (70 - \min (70, F))) + \min (70, F) \right)
\]

\[
= 0.02375 \left( \min (0.5s(100 - F), (70 - \min (70, F))) + \min (70, F) \right)
\]

For our example parameter values her damages can be no higher than $70 and the probability of liability is .02375. Thus, the highest possible expected value for her damages is $70 \times 0.02375 = 1.6625$, which is less than the private benefit she receives from taking the alternative project. So, under Cov, FD if offered project 1 she will take it.\(^7\)

\(^7\)Clearly this is not, in general, the case. These parameter values were chosen to focus in the differential affect of Cov and FD rules on the incentive to undertake project 2 if it is offered. If the private benefit from an alternative project is lower, and/or the difference in expected payout on the base and expected project is lower and/or, the potential actual damages is higher and/or the presumption embodied in the BJR is weaker, then there may well be parameter values that lead to the manager forgoing project 1 if it is offered.
6.2.2. Choice if Offered Project 1, Cov protection for Senior Claim

Her expected total damages if she is offered project 1 and there is covenant protection for the senior claim is are the expected damages conditional on the low payout times the probability of getting the low payout from project 1:

\[
(\Theta - \delta)0.475 \left( \min(0.5s(100 - F), (70 - \min(70, F))) + \min(70, F) \right) = 0.02375 \left( \min(0.5s(100 - F), (70 - \min(70, F))) + .05\min(70, F) \right)
\]

(6.20) \ 
(6.21)

In this case the highest expect damages occur when she has borrowed $70, in that case her expected damages are .05 \times 70 = 3.50. Again, this is less than the value of her private benefit (not to mention the higher incremental payoff from operations) so she takes project 1 if it is offered.

So, for this set of parameter values the liability regime has no effect on the manager's actions if she is offered project 1. As we shall now see, this is not the case if the is offered project 2.

6.2.3. Choice if Offered Project 2, FD protection for Senior Claim

If the manager is offered project 2 her expected income from operations (not including the retention of the collateral, nor her private benefit, is given by:

\[
\begin{pmatrix}
1 - (\Theta - \delta) & (\Theta - \delta) \\
(1 - s)(H - F)
\end{pmatrix}
\begin{pmatrix}
0 \\
(1 - s)(H - F)
\end{pmatrix} = 0.05(H - F)(1 - s)
\]

(6.22) \ 
(6.23)
The incremental expected income from operations is thus

\[ -\delta (H - F)(1 - s) \]  \hspace{1cm} (6.24)

\[ = -0.45 (100 - F)(1 - s) \]  \hspace{1cm} (6.25)

Clearly this depends on the particular values for \( F \) and \( s \), but we can at least ask, for now, how this reduced income compares with the value of the private benefit. We assume that this benefit is given by \( B = 40 \). Thus, the incremental value that she recieves if she takes project 2 rather than the base project is given by:

\[ 40 - 0.45 (100 - F)(1 - s) \]  \hspace{1cm} (6.26)

We present a plot of this function below and the important thing to note is that even before we consider any damages, and the fact that she would get to keep an additional $70 if she stuck with the base project, there are values of \( s \) and \( F \) for which she will not take project 2 if it is offered.

Her expected total damages are the expected damages conditional on the low payout times the probability of getting the low payout from project 2:
\[(\Theta + \delta) 0.475 \left( \min (0.5s (100 - F), (70 - \min (70, F))) + \min (70, F) \right) \]  

\[= (0.95) 0.475 \left( \min (0.5s (100 - F), (70 - \min (70, F))) + \min (70, F) \right) \]  

(6.27)

(6.28)

Recognizing that if she sticks with the base project she gets to keep $70, the change in her net damage related position, retained collateral less expected damages, associated with taking project 2 is given by:

\[70 - (0.95) 0.475 \left( \min (0.5s (100 - F), (70 - \min (70, F))) + \min (70, F) \right) - 70\]

\[= - (0.95) 0.475 \left( \min (0.5s (100 - F), (70 - \min (70, F))) + \min (70, F) \right) \]  

(6.29a)

We present a plot of this function below. Clearly, there are some some combinations of s and F for which the incremental damage payment associated with taking project 2, when coupled with the incremental reduction in expected cash flow to the manager is sufficiently large to offset the private benefit associated taking project 2.

\[\text{In effect, if she sticks to the base project her "damage" payments if } -70.\]
We can explore project choice by looking at the managers project choice. She will take project 2 if value of the private benefit is larger than the sum of the incremental expected cash flows and the incremental damages. That is, she takes the project if:

\[
40 - 0.45 (100 - F) (1 - s) -
\]

\[
(.95) 0.475 \left( \min (0.5s (100 - F), (70 - \min (70, F))) + \min (70, F) \right) > 0 \quad (6.30)
\]

A plot of this function shows that often this is not the case, and she will choose to stick with the base project;
We can define an indicator variable to take the value of 1 whenever she will take project 2 rather than the base project.

\[
i_{2f}(s, F) = \begin{cases} 
1 & \text{if} \\
40 - 0.45125 \min (-0.5s (F - 100), 70 - \min (70, F)) \\
- (s - 1) (0.45F - 45.0) - 0.45125 \min (70, F) > 0 \\
0 & \text{if} \\
40 - 0.45125 \min (-0.5s (F - 100), 70 - \min (70, F)) \\
- (s - 1) (0.45F - 45.0) - 0.45125 \min (70, F) \leq 0 
\end{cases}
\]  
(6.31)
Taking project 2 rather than the base project is a clear manifestation of an agency problem. As expected, we see that the more outside finance the manager has obtained (we ignore, for the time being, the pricing implications of these project choices) the more likely it is that the manager will take the bad project. If she has retained a larger position in the project, by some combination of issuing less debt and retaining a higher fraction of the equity, she is more likely to stay with the base project. Note that for these parameter values the private benefit is less than the incremental reduction in expected cash flows so the project is not first best.

6.2.4. Choice if Offered Project 2, Cov protection for Senior Claim

Covenant protection for the senior claim may lead to different decisions because the expected damage payment is higher under Cov than it is under FD. under Cov protection her expected net total damages as presented in equation 6.29a become:
\[ 70 - (.95) \left( 0.475 \left( \min (0.5s (100 - F), (70 - \min (70, F))) \right) + \min (70, F) \right) - 70 \quad (6.32) \]

\[ = - (.95) \left( 0.475 \left( \min (0.5s (100 - F), (70 - \min (70, F))) \right) + \min (70, F) \right) \quad (6.33) \]

We present a plot of this function below. Here we see that the net expected damages are greater than in the case where the senior claim had only FD protection.

We can compare the expected damages by plotting the net expected damages for both regimes on the same graph. This is done below.
Clearly the expected damages are greater, for these parameter values, when the senior claim has Cov protection. Is the difference sufficient to affect project choice. Again, we can explore project choice by looking at the managers project choice. She will take project 2 if value of the private benefit is larger than the sum of the incremental expected cash flows and the incremental damages. That is, she takes the project if:

$$40 - 0.45 (100 - F) (1-s) - 0.95 \times 0.475 (\min(0.5s (100 - F), (70 - \min(70, F)))) - 0.95 \min(70, F) > 0$$

(6.34)

A plot of this function shows that often this is not the case, and she will choose to stick with the base project;
It appears clear that there are many more combinations of $F$ and $s$ for which the manager will choose to undertake a suboptimal project choice. Again can define an indicator variable to take the value of 1 whenever she will take project 2 rather than the base project.

$$
i2c(s, F) = \begin{cases} 
1 & \text{if} \quad 0.45 \min (0.5s (100 - F), (70 - \min (70, F))) \\
40 - 0.45 (100 - F)(1 - s) - & \min (70, F) > 0 \\
-0.95 \min (70, F) & \min (70, F) \leq 0 \\
0 & \text{if} \quad 0.475 \min (0.5s (100 - F), (70 - \min (70, F))) \\
40 - 0.45 (100 - F)(1 - s) - & \min (70, F) > 0 \\
-0.95 \min (70, F) & \min (70, F) \leq 0 
\end{cases} \quad (6.35)$$
From this plot we see that when the bondholders have the same claim to payment from cash flows, but are protected against the manager choosing the inferior project by contractual protection rather than a fiduciary duty there many more combinations of debt and outside equity that lead to "good" managerial behavior. Indeed, for these parameter values the manager will make the right decision any time he has retained more than 80% of the equity, no matter how much he borrowed.

6.3. Financing

The potential investors will clearly recognize the manager’s incentives to take on a bad project in order to get the private benefit. They will price each security by the expected payoff that it affords (recall that by assumption all investors are risk neutral and the risk free interest rate is 0). Because, in expected value terms, the damages are insufficient to make the investors whole in the even that a manager, offered project 2 takes it rather than the base project, it must be the case that the manager’s choice of financing affects the amount of money he can raise. Depending on the cost of the project the manager may find herself either unable to raise sufficient funds for the project, or only able to do so by giving up an unacceptably large percentage of the revenues from the project.
This latter would become a consideration if the manager has an opportunity cost for her time so that simply raising money for the project does not mean that the manager is interested in doing it.

6.4. The BJR again

We now return to the bjr presumption that, under certain circumstances, the board is protected from second guessing by the court. We have captured this by positing a "thumb on the scale" that reduces the probability of liability under FD standards. In our example that factor was .5. Consider the situation where there is a much, much stronger BJR at work. Say the factor is .01. We turn to our analysis of the managers choice under FD, FD when offered project 2. Expected damages now fall to (.95) 0.0095 \left( \min (0.5s (100 - F), (70 - \min (70, F))) + \min (70, F) \right). The indicator variable for whether the manager takes project 2 when it is offered now becomes:

\[
i2f(s, F) = \begin{cases} 
1 & \text{if } 40 - 0.009025 \min (-0.5s (F - 100), 70 - \min (70, F)) \leq 0 \\
-s - 1 (0.45F - 45.0) - 0.009025 \min (70, F) > 0 \\
0 & \text{if } 40 - .009025 \min (-0.5s (F - 100), 70 - \min (70, F)) \leq 0 \\
-(s - 1) (0.45F - 45.0) - .009025 \min (70, F) \leq 0 
\end{cases}
\]

We plot the variable below.
Here we see that for virtually any amount of outside financing the manager will choose the bad project if it is presented. This means that a manager with limited or no funds to invest in the project may have difficulty raising sufficient funds to undertake it. There will also be some effect in the Cov, FD regime, but it will not be nearly as great. In the FD,FD regime a stronger standard for finding a lack of BJR protection reduces the likelihood that value of both debt and equity will success in litigation against the manager, while in the Cov, FD regime the change only affects the equity to collect.

7. Summary and Intuition

In this model, as in many others, choice of capital structure helps to determine the project choice that managers make when faced with new investment opportunities. Debt serves as a commitment mechanism that will induce managers not to deviate from the production plans used to arrange the initial financing. For our simple example, where the real fear is the private benefit that managers obtain from the alternative projects may be sufficient to induce managers to take projects whose cash flow alone is insufficient to cover their costs, we see that there is an \textit{ex ante benefit} when
bondholders are protected by an indenture, rather than by fiduciary duties. We also an interesting, and perhaps heretofore not realized, connection between legal regime an the Business Judgement Rule. If we are to have a meaningful BJR, the thumb on the scale must be rather heavy. However, in that case, if debtholders had FD rather than Cov protection, managers would have no way to offer outside investors a sufficiently strong commitment to good behavior. The ability to write a restrictive bond indenture, then, serves to facilitate raising funds by entrepreneurs.

Of course, Cov protection for bondholders comes at a cost. In our example the difference in expected cash flow between the base project and the alternative projects was rather large. In other words, the value of flexibility was large. One can easily imagine situations where, say the private benefit from the alternative projects was not so large, and managers might decline to take even the good alternative project under Cov protection. This would be more likely to be true if we imagine that the manager, who, may well be less diversified than the outside investors, is risk averse.