Why Divorce Laws Matter: Incentives for Non-Contractible Marital Investments under Unilateral and Consent Divorce\textsuperscript{1}

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Abstract

The Coase Theorem suggests that married couples will divorce if and only if doing so increases their joint surplus, regardless of the legal rules governing divorce. This does not mean, however, that divorce laws only affect the distribution of rents. Because the distribution of rents affects each spouse’s incentives for non-contractible investments, divorce laws can affect the joint welfare of the couple. This paper analyzes the effects of the consent divorce regime and the unilateral divorce regime on incentives for selfish and cooperative marital investments. Using these results, the paper demonstrates how endogenous choice of marriage with non-contractible investments can explain some recent empirical results concerning the effects of the shift from consent divorce to unilateral divorce. Finally, the paper explores why making divorce costly (but not too costly) can increase the joint welfare of a couple.
1 Introduction

Few laws affect as many people in as significant a way as laws governing marriage and divorce. In the 1970s, divorce laws in the United States began to undergo a radical change. Prior to 1969, all states operated under the consent divorce regime: to obtain a divorce both spouses had to consent, unless one spouse could demonstrate "fault" by the other spouse. In 1969, California was to the first state to adopt a "no fault" divorce rule. During the 1970s, almost every state followed California’s lead and changed their divorce law from the consent divorce regime to the unilateral divorce regime. Now, either spouse has the option of obtaining a divorce without the consent of his or her partner.

Becker (1981) and Becker et. al. (1997) argue that the Coase Theorem applies to marital bargaining, thus, regardless of the divorce regime, divorces should occur if and only if they increase the joint surplus of the couple. Changing from the consent divorce regime to the unilateral divorce regime might affect the distribution of rents between the spouses, but, according to this theory, this change should not have affected the overall divorce rate.\(^1\) The empirical evidence, however, contradicts this view. Wolfers (2003) has found both that the move from consent divorce to unilateral divorce increased the divorce rate in the short run but that, in the long run, the divorce rate is no higher (and may even be slightly lower) than what it would have been under consent divorce.\(^2\)

In this paper, I develop a model that can explain both the initial positive effect on the divorce rate after the shift to unilateral divorce and the subsequent negative effect on the divorce rate. This model also demonstrates the efficiency advantages and disadvantages of the two regimes.\(^3\) What drives the results in this paper is the assumption that a spouse can make relationship-specific investments in the marriage and that these investments are non-contractible. Much of the effort that goes into making a marriage work is relationship-specific. Spending less time at work (and thus reducing one’s chances for personal success) and more time with your current spouse can often improve the bond with your current spouse but is of no value if you obtain a divorce. Similarly, learning to tolerate your partner’s foibles (or enjoy her hobbies) or trying to change your own habits that annoy your spouse will pay dividends only if you stay together. Spending time with your

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\(^1\)The divorce rate is the number of new divorces per thousand people.

\(^2\)Mechoulan (2005) also finds a similar pattern in divorce rates in response to the change in divorce regimes.

\(^3\)In this paper, by efficiency I mean the joint surplus of the couple. I do not include children in the model. Considering the effect of divorce rules on incentives for investment in children is an important avenue for future research. For an empirical analysis of the effect of unilateral divorce on children, see Gruber (2004).
partner’s friends and family or changing your parenting style to match your spouse’s also have large relationship-specific components.

While it is reasonable to assume (as this paper does) that spouses can bargain efficiently over whether or not to divorce, it is hard to imagine that they can write binding contracts over all possible investments that each spouse could make that affect the relative value of the marriage. To the extent that these investments are non-contractible, divorce rules that affect the division of rents between the spouses will also affect incentives for non-contractible investments. This paper analyzes the effect that these two divorce regimes have on incentives for different types of non-contractible investments. In addition, it also compares incentives for relationship-specific investments under unilateral divorce with the incentives for such investments among unmarried couples.

Why might marital investments not be contractible? In many cases, the precise duties may be hard to describe or verify. How would a court determine whether I’ve really made a substantial effort to be a better bridge partner for my wife or (even more difficult) to be more considerate? Since spouses often share money, the appropriate form of compensation may also be hard to describe. The very act of compensating such investments (or even writing the contracts themselves) may be quite costly in that it may reduce one or both spouses’ utility from the marriage (asking for a contract or compensation will often be seen as unromantic). Lastly, in many cases it seems likely that a court would refuse to enforce such a contract between spouses.

I consider two different types of non-contractible investments in marriage: selfish relationship-specific investments and cooperative relationship-specific investments. Any type of investment a spouse can make that affects the value of a marriage is a linear combination of these two types of investments. A selfish relationship-specific investment is an investment that affects the investing spouse’s utility from the marriage but does not affect her partner’s utility from the marriage. A cooperative relationship-specific investment affects only the non-investing spouse’s utility from the marriage. While I analyze the effect of the divorce regime (either consent or unilateral) on each of these extreme types of investments, in reality, many marital investments will have features of both. Nonetheless, the results in this paper are instructive for these investments as well since

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4 A previous version of the paper also considered selfish outside option investments and cooperative outside option investments.

5 If one spouse learns to play bridge so as not to be left out when the other spouse goes to bridge tournaments, this may be primarily a selfish relationship-specific investment. Of course, the other spouse’s utility from this marriage is likely to be different when her spouse is joining her at bridge tournaments (it could be higher or lower), so this investment has cooperative features as well.
they demonstrate how each divorce regime affects the incentives for the selfish and cooperative components of any given investment.

Consent divorce and unilateral divorce have very different effects on selfish and cooperative investments for two, interrelated, reasons. First, the different default rules change the direction of the compensation in marital bargaining. Under consent divorce, the default rule (what happens unless both sides agree to change it) is that a married couple stays married. Thus, if one spouse wants a divorce and the other does not, the spouse that wants a divorce has to compensate the other to obtain a divorce. By contrast, under unilateral divorce, the default is divorce; the spouse who wants to stay married now has to compensate the other spouse to prevent a divorce. Second, the two regimes create two different types of bargaining games. When spouses bargain under the consent divorce regime, they are bargaining to end a marriage. The alternative to reaching an agreement is to stay married, which does not require any action by either spouse. So, in the language of bargaining theory, staying married is a "disagreement payoff" or "inside option." It determines each spouse’s utility while bargaining and if bargaining breaks down. On the other hand, when spouses bargain under the unilateral divorce regime, they are bargaining to save the marriage. The alternative to reaching an agreement is getting a divorce. In this case, however, getting a divorce requires an action by one spouse that ends the bargaining. So, getting a divorce is an "outside option," it determines each spouse’s utility if bargaining breaks down but does not affect their utility while bargaining is proceeding. This difference is quite important in determining how the alternative option affects the bargaining equilibrium, which, in turn, leads to significant differences in investment incentives.

The main findings are as follows. Unilateral divorce can lead to insufficient incentives for selfish relationship-specific investments because such investments sometimes just reduce the amount non-investing spouse will have to compensate the investing spouse to remain in the marriage. On the other hand, under consent divorce these incentives can be excessive. Even if the investing spouse suspects the marriage will end in divorce, improving her marriage utility gives her a stronger bargaining position when the couple bargains over divorce.

The findings for cooperative relationship-specific investment are nearly the opposite. While incentives under unilateral divorce are often insufficient, unilateral divorce does provide some incentives for cooperative investments (and even efficient incentives in some circumstances). Under consent divorce, by contrast, a spouse never has an incentive to improve her partner’s marriage utility and often has an incentive to decrease it. The reason for the difference is that under uni-
lateral divorce, one partner often has to compensate the other to stay married. The greater that spouse’s marriage utility, the less the other spouse must compensate him to stop him from asking for a divorce. Under consent divorce, a spouse that wants to stay married can do so without compensating the other spouse at all. These findings are consistent with recent empirical evidence on the effect of the divorce regime on rates of spousal abuse. Wolfers and Stevenson (2003) have found that the shift to unilateral divorce has led to a significant reduction in spousal abuse. Since spousal abuse reduces one’s partner’s utility from the marriage, investments a spouse makes to reduce his likelihood of abusing his spouse (like going to counseling) are cooperative investments. These are exactly the types of investments that unilateral divorce laws encourage but consent divorce laws discourage.

This paper also develops a model of endogenous marriage to show how the change in the divorce regime will change both the marriage decision and the investment decision in a way that creates a change in the divorce rate that is consistent with Wolfers’ (2003) finding. The key to this result is that different types of investments are likely to be important in different relationships, and people have an incentive to choose a relationship where the important investments are of the type that the legal regime encourages. As a result, if there is an unanticipated shift in the legal regime, these marriages will be less likely to survive. To be more concrete, under the consent regime, ceteris paribus, one should always choose a marriage where selfish investments are likely to be important rather than a marriage where cooperative investments will be important (even if the cooperative investments are much more productive) since consent divorce provides no incentives for cooperative investments. When the legal regime changes to unilateral divorce, there is now less incentive for spouses in this type of marriage to make these investments. This increases the probability of divorce. This effect, however, is naturally temporary as couples that get married under the new legal regime choose their relationships with knowledge of the different types of investments the new regime supports. That is, once people realize the divorce regime will be unilateral divorce, they may now choose marriages where cooperative investments are important. Under certain conditions, they will choose the cooperative marriage precisely when that marriage has a lower probability of divorce. Thus, once people expect unilateral divorce, people can more effectively choose their marriages in a way that lowers the risk of divorce. Whether the final divorce rate ends up higher or lower than what it was under consent divorce depends on how frequently marriages with very productive cooperative investment opportunities arise. If they arise quite frequently, then the steady state divorce rate under unilateral divorce could be as low or lower than the steady
state divorce rate under consent divorce, which is exactly what the empirical evidence indicates.

Determining the optimal divorce regime, however, involves more than just deciding whether one should require the agreement of one or both spouses to allow a divorce. Another important issue is how easy should it be for a couple to obtain a divorce (independent of the number of spouses who want it). If obtaining a divorce is costless, then (under unilateral divorce) a marriage is not very different from an unmarried relationship. In fact, two states, Arizona and Louisiana, have recently passed "covenant" marriage laws. These laws make divorce more costly for both spouses by requiring extensive counseling prior to divorce, though they still allow for unilateral divorce. Seventeen other states have considered similar laws. I also analyze this question of whether it can ever make sense to increase the cost of divorce under the unilateral divorce framework where both spouses can make investments. I show that making divorce costly, but not too costly, can induce efficient relationship-specific investments (both selfish and cooperative) that unmarried couples would not make. This is another example of committing to an ex post inefficient action in order to induce more efficient ex ante decisions.

This paper follows a long line of literature that considers the two spouses in a marriage as two individuals with separate utility functions. Manser and Brown (1980) and McElroy and Horney (1981) were the first to consider a household as having two members with potentially competing interests and to explicitly model bargaining within marriage. Lundberg and Pollack (1993) then analyzed household bargaining over the provision of a public good for the household where the threat points were given by a non-cooperative equilibrium within the marriage rather than by the spouses' divorce utilities. Lundberg and Pollack (1996) provide an excellent overview of the literature on decision making within a marriage.

Two recent papers, Clark (1999) and Fella et. al. (2004), present models that show that if utility within a marriage is not fully transferrable between the spouses, then divorce laws can affect the equilibrium divorce rate. Since neither of those papers consider the issue of investment incentives within a marriage, but my paper does not consider non-transferrable utility, those two papers and this one provide complementary reasons why divorce laws matter. Some papers have explicitly dealt with investment incentives in marriage. Dufwenberg (2002) and Konrad and Lommerud (2000) both analyze the efficiency of non-contractible investments in education but do not consider the effect of divorce laws. Peters (1986) mentions that moral hazard may lead to inefficient marital investments, but her focus is on empirically estimating the effect of the change in legal regime on divorce rates, so she does not provide a formal model of non-contractible marital investments.
Brinig and Crafton (1994) argue that the change to unilateral divorce has undermined incentives for marriage-specific investments, but they do not distinguish between different types of investments and do not present a formal model of the incentive effects of the different legal regimes. Matouschek and Rasul (2004) model different theories of the role of the divorce contract and test those theories assuming that the change from consent to unilateral divorce lowers the cost of divorce. They do not, however, consider the direct effect of the legal change on the incentive for non-contractible investments in marriage.

Finally, there is a literature on the marriage market that bears some relation to this paper. Chiappori and Weiss (2000) present a general equilibrium model of divorce and remarriage that focuses on contracting externalities, rather than non-contractible investments, to show that there may be too little divorce in equilibrium. Peters and Siow (2002) analyze the effect of marriage market competition on parental incentives for pre-marriage investments in their children’s wealth, but they do not consider investments by the spouses themselves that affect the value of their marriage.

The plan of the paper is as follows. Section 2 presents the model of one-sided non-contractible investments with spousal bargaining under both consent and unilateral divorce laws. Subsection 2.1 analyzes selfish relationship-specific investments; 2.2 analyzes cooperative relationship-specific investments. Section 3 describes the endogenous marriage model (subsection 3.1) and discusses how these results relate to recent empirical findings on the effect of unilateral divorce laws on the divorce rate and rates of spousal abuse and suicide (subsection 3.2). Section 4 introduces a two-sided investment model to analyze the effects of making (unilateral) divorce costly on the incentives for selfish relationship-specific investments (subsection 4.1) and cooperative relationship-specific investments (subsection 4.2). Section 5 concludes.

## 2 One-Sided Investment Model

Consider a marriage between two people, one of which has the opportunity to make a non-contractible investment\(^6\) that costs her \(c\) (I’ll use the female pronoun for the potential investor and the male pronoun for the non-investor). If she does not invest, then the potential investor (hereafter, just the investor, even if she does not invest) gets a utility of \(u_I\) from the relationship

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\(^6\)In a bilateral investment model, the investments would be strategic complements, so there would be equilibria where one spouse would invest if the other spouse is going to invest also. This would not change the nature of the comparison between unilateral and consent divorce.
and the non-investor gets utility $u_N$ from the relationship.\textsuperscript{7} If she does invest, then her ex post utility is $u_I + b_I$ (her ex ante utility is $u_I + b_I - c$) and her spouse's utility is $u_N + b_N$. I normalize each person's utility if the marriage dissolves to zero.\textsuperscript{8}

The timing of the game is as follows. In stage one, the investor chooses whether to invest. In stage two, the two parties observe their payoffs from staying married and getting divorced and engage in an infinite horizon, alternating offer bargaining game. If the law allows unilateral divorce, then in any given period of the bargaining game, one party offers to give the other party a transfer (possibly negative) in exchange for a promise to stay married. Note, this transfer need not be monetary. Rather than one spouse giving another spouse money, that spouse could make cooperative investment that has the same effect as a monetary transfer.\textsuperscript{9} The other party either accepts, rejects, or obtains a divorce. If the responder rejects, then each party obtains its marriage utility for that period and in the next period the responder makes an offer. Of course, promises to stay married are not actually binding, but the transfer can be paid out over time such that at any time the promise of the future stream of payments is such that cost of reneging on this promise is the same as the cost of not making it in the first place. This makes the promise self-enforcing.

\section*{2.1 Selfish Relationship-Specific Investments}

\subsection*{2.1.1 Unilateral Divorce}

If investments are purely selfish, then $b_N = 0$. In this case, one can restrict attention to investments where $b_I > 0$. Because, with unilateral divorce, either spouse can obtain a divorce without the consent of the other spouse, the equilibrium of the stage 2 bargaining game is a straightforward extension of Shaked and Sutton's (1984) model of a Rubinstein (1982) bargaining game with outside options. I will focus on the limiting case where the discount factor approaches one. In this case, the outside option is divorce. As Shaked and Sutton have shown, outside options have no effect on the equilibrium unless they are greater than one player's equilibrium payoff (in which case, the

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\textsuperscript{7}In this section, marriage utilities are deterministic. That is, they reflect the utilities at the time the investment decision is made. But, since I consider all possible combinations of marriage utilities, the results are instructive for situations where future marriage utilities are uncertain. In the next section, I explicitly model the fact that future marriage utilities are uncertain at the time of marriage.

\textsuperscript{8}Note, each spouse maximizes her own utility and does not directly consider the utility of her spouse in making decisions. This does not completely rule out altruistic preferences, however, in that one spouse's own utility could depend on the utility of the other spouse. That said, if the spouses had altruistic preferences, then there would be no such thing as a purely selfish or purely cooperative investment. Of course, with altruistic preferences, both spouses would bear the cost of the investment as well.

\textsuperscript{9}This does not necessitate that the investment is contractible. This investment is made immediately, so it does not require any court enforcement.
outside option is binding). Since the couple is already married, if neither has a binding outside option, neither has a credible threat that enables her to extract a payment from the other. Thus, the parties will just receive their marriage utilities. If one spouse (say the wife) does have a binding outside option, as long as marriage still maximizes joint surplus, the husband compensates his wife so that her payoff from marriage equals her payoff from exercising her outside option (which I have normalized to zero). If divorce maximizes joint surplus, then there is nothing to bargain over, so each spouse obtains a payoff of zero.

Let \( \theta \) be an indicator variable that equals one if the investor invested and zero if she did not. Notice that if \( u_I + \theta b_I, u_N > 0 \), then neither party has a credible threat to divorce, so neither party has to pay the other to stay married. The equilibrium is marriage with no transfer, and the period two payoffs are \( u_I + \theta b_I, u_N \). If \( u_I + \theta b_I + u_N < 0 \), then there is no transfer that can prevent a divorce. So, the equilibrium is divorce, and the period two payoffs are 0,0. If \( u_I + \theta b_I + u_N > 0 \), but either \( u_I + \theta b_I < 0 \) or \( u_N < 0 \), then the party with the negative payoff from the marriage has a credible threat of divorce. Thus, Shaked and Sutton’s outside option principle implies that the equilibrium is marriage and the spouse with the positive payoff must give the other spouse a transfer that brings her total payoff up to zero. These equilibria and the associated payoffs are summarized in the following table.

<table>
<thead>
<tr>
<th>Case</th>
<th>Conditions</th>
<th>Outcome</th>
<th>Gross Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marital Bliss</td>
<td>( u_I + \theta b_I, u_N &gt; 0 )</td>
<td>Marriage</td>
<td>( u_I + \theta b_I, u_N )</td>
</tr>
<tr>
<td>( I ) Tolerates ( N )</td>
<td>( u_I + \theta b_I + u_N &gt; 0, u_I + \theta b_I &lt; 0 )</td>
<td>Marriage</td>
<td>( 0, u_I + \theta b_I + u_N )</td>
</tr>
<tr>
<td>( N ) Tolerates ( I )</td>
<td>( u_I + \theta b_I + u_N &gt; 0, u_N &lt; 0 )</td>
<td>Marriage</td>
<td>( u_I + \theta b_I + u_N, 0 )</td>
</tr>
<tr>
<td>Divorce</td>
<td>( u_I + \theta b_I + u_N &lt; 0 )</td>
<td>Divorce</td>
<td>0,0</td>
</tr>
</tbody>
</table>

Now consider \( I \)'s investment decision. If there will be Marital Bliss even if she does not invest, then investing increases \( I \)'s net payoff by \( b_I - c \), which provides socially efficient incentives to invest. The other easy case is where there is divorce whether or not \( I \) invests, then \( I \) does not invest, which is obviously socially efficient.

If there would be divorce if \( I \) does not invest, then investing could move the stage 2 bargaining game into any of the other three cases.\(^{10}\) If the investment moves the couple to \( N \) Tolerates \( I \),

\(^{10}\)If \( N \) has a positive marriage utility, then \( I \)'s investment could give her a positive marriage utility also, moving the couple into Marital Bliss. Alternatively, \( I \)'s investment could increase her marriage utility enough to make the joint surplus from marriage positive even if her marriage utility is still negative. This is where investment moves
then investing does not change $N$’s gross payoff, thus $I$’s gain from investing is exactly the social gain, giving her efficient incentives. When $I$ invests, she increases her payoff from the marriage, but this does not affect what she has to pay her spouse to stop him from getting a divorce. This payment is fixed at her spouse’s loss from staying married, which causes the investor to internalize all the benefits and costs of marriage. The same argument shows that when the couple would be in $N$ Tolerates $I$ without investment, then the investing spouse captures all the benefits from investment, giving her efficient incentives.

On the other hand, if the investment moves the couple to $I$ Tolerates $N$, then investing does not change $I$’s gross payoff. So, $I$ has no incentive to invest at all. The social gain from investing $(u_I + b_I + u_N)$, however, is positive. $I$ has an insufficient incentive to increase her marriage utility because this just reduces the amount that her spouse has to give her to induce her not to get a divorce. While the investment is selfish in that it does not change the spouse’s payoff from the marriage, its effect, due to the nature of outside option bargaining, is purely selfless. There will not be investment when it is socially efficient if and only if $c < u_I + b_I + u_N$.

If the investment moves the couple from divorce to Marital Bliss, then both spouses’ gross payoffs increase. Thus, while $I$’s ex post gain from investing is $u_I + b_I$, the social gain is $u_I + b_I + u_N$. This exceeds $u_I + b_I$ since $u_N > 0$ in Marital Bliss. Because the non-investing spouse gains from the marriage, investing and saving the marriage has positive external effects. There will not be investment when it is socially efficient if and only if $u_I + b_I < c < u_I + b_I + u_N$.

If the marriage would be one where $I$ Tolerates $N$ without investment, there will also be insufficient investment incentives. If investment does not change the case ($b_I < -u_I$), then $I$’s investment does not increase her gross payoff at all; it only reduces the transfer from $N$. Once again, the selfish investment turns out to be purely selfless, giving $I$ no incentive to invest, even if the investment is socially efficient, $c < b_I$. If the investment moves the couple to Marital Bliss, then $I$ does receive some of the benefit from her investment $(u_I + b_I)$, but this is less than the social benefit $(b_I)$ because investing also eliminates the need for the non-investing spouse to compensate the investing spouse to induce her to remain in the marriage. So, there is no investment even though it is socially efficient if and only if $u_I + b_I < c < b_I$.

Because investments in marital utility are non-contractible, an investing spouse will often not be able to capture all of the surplus from her investment, giving her insufficient incentives to make the couple to $I$ Tolerates $N$. Lastly, if $N$’s marriage utility is negative, $I$’s investment could increase her marriage utility enough to make the total marriage surplus positive, moving the couple to $N$ Tolerates $I$. 

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such investments. The reason for this, however, is somewhat different from the standard holdup argument. In the standard holdup story, non-contractible investments increase the surplus from trade. Since bargaining over this surplus occurs after the investments are made, the non-investing party gets a share of the surplus. With marital investments, however, the relationship contract is already set. The ex post bargaining is now over whether to continue the relationship or not. So, if the investing spouse was unhappy with the relationship before her investment, her investment then decreases or eliminates the amount the other spouse has to compensate her to stay in the marriage. Thus, some (or all) of the benefit from the investment accrues to the non-investing spouse. This is the source of the under-investment problem in this setting.

2.1.2 Consent Divorce

When divorce requires the consent of both parties, the bargaining game between the two spouses is quite different. Instead of bargaining to prevent one partner from exercising her outside option of divorce, the spouses are bargaining to obtain consent for a divorce. This means that neither side has an outside option. The parties obtain their marriage payoffs until an agreement is reached. Thus, it follows from Rubinstein (1982) that as the discount factor approaches one, the solution to the bargaining game is given by the Nash bargaining solution if divorce increases joint surplus. If divorce does not increase joint surplus, then there is no scope for bargaining, each spouse just obtains her marriage utility. The equilibria and the associated payoffs are summarized in the following table.

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<td>$u_I + \theta b_I + u_N &gt; 0$</td>
<td>Marriage</td>
<td>$u_I + \theta b_I, u_N$</td>
</tr>
<tr>
<td>Divorce</td>
<td>$u_I + \theta b_I + u_N &lt; 0$</td>
<td>Divorce</td>
<td>$(u_I + \theta b_I - u_N)/2, (u_N - u_I - \theta b_I)/2$</td>
</tr>
</tbody>
</table>

If divorce can increase joint surplus, then the parties will agree to it. Under the Nash bargaining solution with equal discount factors, each party obtains their disagreement payoff (marriage) plus half the surplus from the agreement (eliminating the net loss from marriage). Since each party obtains her marriage utility when marriage is the ex post efficient outcome, if marriage is efficient without investment, then, because the investment is selfish, the investing spouse receives the entire surplus from her investment, making investment incentives socially optimal. If there would be divorce without investment, however, then the consent regime actually gives excessive incentives.
for selfish relationship-specific investments. The reason is that under the consent regime, one’s payoffs when a divorce happens are greater the greater is one’s marriage utility. Even if investing in the marriage does not save the marriage, it improves the investor’s bargaining position when the spouses are bargaining to achieve a divorce. Her payoff in the divorce regime is $b_I/2$ greater when she invests than when she does not. Thus, the investing spouse has an incentive to increase her marriage utility even though this is socially useless since the marriage will dissolve anyway.\textsuperscript{11}

In fact, the investing spouse’s incentive to make selfish relationship-specific investments is excessive even when this investment saves the marriage. To see this, subtract her payoff when there is marriage with investment from what it would be with divorce without investment. This difference is $b_I + (u_I + u_N)/2$. The social value of this investment, however, is that it saves the marriage, moving the total surplus from zero to $b_I + u_I + u_N$. Since there would be a divorce absent the investment, $u_I + u_N < 0$, this implies that the private incentive to make these investments exceeds the socially optimal level.

The following proposition summarizes the results of this subsection.

**Proposition 1–Selfish Investments**

A. Under unilateral divorce, private incentives to make selfish relationship-specific investments are socially optimal if there is Marital Bliss without the investment, if there will be divorce even with the investment, or if the marriage is in case $N$ Tolerates $I$ with the investment. In any other situation, unilateral divorce gives a spouse strictly too little incentive to make selfish relationship-specific investments.

B. Under consent divorce, if the marriage will survive without the investing spouse making a selfish relationship-specific investment, then incentives for selfish relationship-specific investments are socially efficient. Otherwise, this incentive is socially excessive under the consent regime.

This proposition tells us that, if the only important investments were selfish ones, then the consent regime would give spouses a greater incentive to make investments to save their marriages than does the unilateral divorce regime. In fact, while the incentives in the unilateral divorce regime are often insufficient, the incentives in the consent regime are often excessive.

\textsuperscript{11}For simplicity, I will often just analyze the incentives to make investments, that is the private gain from the investment. Obviously, having a positive incentive to make an investment does not mean it will occur. But if one’s incentive is less than (greater than) the socially optimal incentive, then there is a region of costs for which the spouse will not (will) invest even though it is socially optimal to (not) invest. Furthermore, if the magnitude of the gain were an increasing and concave function of the amount spent, then differing private and social incentives to invest would necessarily imply suboptimal investment levels.
2.2 Cooperative Relationship-Specific Investments

2.2.1 Unilateral Divorce

The last subsection considered investments that only affected the investing spouse’s marriage utility. This section considers the polar opposite case of investments that only affect the marriage utility of the non-investing spouse. Thus, in this subsection, \( b_I = 0 \). I do not assume that \( b_N \) is necessarily positive, however, to allow for the fact that one spouse may have an incentive to decrease the marriage utility of her partner.

Because ex post utilities, and not how those utilities came about, are all the matter for the bargaining outcome, the equilibria and the payoffs associated with them are identical to what they were in Table 1, except that the investment payoff term now accrues to \( N \) rather than \( I \).

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</tbody>
</table>

One can now determine \( I \)’s incentive to make cooperative investments from Table 3. If, without investment, the marriage is in Marital Bliss or \( I \) Tolerates \( N \), then \( I \) has no incentive to make cooperative investments. Raising \( N \)’s marriage utility in these states does not increase \( I \)’s payoff. If, however, \( u_I > 0 \) and \( u_N < 0 \), then \( I \) will capture some or all of the surplus her investment creates. This can happen either when the no-investment outcome is \( N \) Tolerates \( I \) or Divorce. Unless the investment moves the marriage all the way to Marital Bliss, this investment in increasing \( N \)’s marriage utility does not affect his payoff at all. Thus, \( I \)’s benefit from this investment is exactly the social benefit. If the investment moves the marriage from Divorce to Marital Bliss, then \( I \)’s payoff increases from zero to \( u_I > 0 \) and \( N \)’s payoff increases from zero to \( u_N + b_N > 0 \). Similarly, if this investment moves the marriage from \( N \) Tolerates \( I \) to Marital Bliss, then \( I \)’s payoff increases from \( u_I + u_N \) to \( u_I \), but \( N \)’s payoff also increases from zero to \( u_N + b_N > 0 \). In these cases, \( I \) has some incentive to make cooperative investments, but her incentive falls short of the social benefit from her investment. Lastly, notice that \( I \) clearly can never gain from making an investment with \( b_N < 0 \).
Cooperative investments are notoriously hard to motivate (Che and Hausch 1999). This is especially true where the terms of the relationship are already fixed (as they are in marriage). Unilateral divorce, however, can motivate these investments by giving the non-investing spouse an outside option of divorce. If the investing spouse wants to avoid divorce, her investment in making her partner happier with the marriage makes it easier for her to convince him not to ask for a divorce. Essentially, cooperative investments decrease the relative value of the non-investor’s outside option (divorce) versus staying with the relationship (marriage). Where that outside option is binding, investment incentives are efficient. If the investment moves this outside option from binding to non-binding, there is some incentive to invest, though it is socially insufficient. While, if the option is never binding, the investing spouse receives no benefit from reducing its relative value.

2.2.2 Consent Divorce

As in the prior subsection, the bargaining outcome with cooperative investments under the consent regimes will look very similar to the outcome with selfish investments; one just attributes the value from the investment to \( N \) instead of \( I \). Thus, we have the following table:

<table>
<thead>
<tr>
<th>Case</th>
<th>Conditions</th>
<th>Outcome</th>
<th>Gross Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marriage</td>
<td>( u_I + u_N + \theta b_N &gt; 0 )</td>
<td>Marriage</td>
<td>( u_I, u_N + \theta b_N )</td>
</tr>
<tr>
<td>Divorce</td>
<td>( u_I + u_N + \theta b_N &lt; 0 )</td>
<td>Divorce</td>
<td>( (u_I - u_N - \theta b_N)/2, (u_N + \theta b_N - u_I)/2 )</td>
</tr>
</tbody>
</table>

If the couple will stay married even without an investment, then the investing spouse has no incentive to make a cooperative relationship-specific investment since it will not affect her payoff. If the couple will get a divorce regardless of the investment, then the investing spouse’s payoff if she invests is decreasing in \( b_N \). Thus, she will not make a positive investment (which is efficient, since it does not prevent the divorce), but she does has an incentive to make a negative investment. If she can reduce her partner’s marriage utility, then she reduces his disagreement payoff, which increases her payoff.

If the investment saves the marriage, then the investing spouse’s payoff goes from \( (u_I - u_N)/2 \) with a divorce to \( u_I \) with the marriage. This changes her payoff by \( (u_I + u_N)/2 \). This is negative since without the investment the joint surplus from the marriage is negative. Thus, the investing spouse has no incentive to make a cooperative investment that would save the marriage. Now
consider a negative cooperative investment that causes the marriage to end in divorce. This changes the investing spouse’s payoff from \( u_I \) to \( (u_I - u_N - b_N) / 2 \). This changes her payoff by \(- (u_I + u_N + b_N) / 2 \). This is positive since this negative investment causes a divorce, so the joint surplus from marriage is negative. The consent divorce regime gives a spouse the incentive to make negative cooperative investments that lead to divorce. Clearly, this is socially inefficient.

The following proposition summarizes the results of the cooperative investments subsection.

**Proposition 2—Cooperative Investments**  
A. Under the unilateral divorce regime, a spouse has no incentive to make cooperative relationship-specific investments unless the investing spouse has a positive marriage utility while the non-investing spouse’s marriage utility is negative. In this case, the incentive to make cooperative relationship specific investments is socially efficient unless this investment moves the marriage to Marital Bliss. If the investment does move the marriage to Marital Bliss, then there are positive, but socially insufficient incentives for cooperative relationship-specific investments.

B. Under the consent divorce regime, there is no incentive to a make cooperative relationship-specific investment that increases the marriage utility of one’s partner, even if this investment is socially efficient. On the other hand, a spouse does have an incentive to make a cooperative relationship-specific investment that reduces the marriage utility of one’s partner if this investment causes a divorce or if there would be a divorce with or without this investment.

Under the consent regime, one spouse does not have to compensate an unhappy spouse to prevent a divorce. This eliminates the incentive to increases the other spouse’s marriage utility that is present in the unilateral divorce regime. Moreover, that fact that spouses have to bargain for a divorce means that the lower one spouse’s marriage utility, the more he is willing to transfer to his partner to get her to agree to a divorce. This gives the investing spouse an incentive to lower her partner’s utility if she anticipates that a divorce will occur. This perverse incentive does not exist in the unilateral divorce regime. Thus, to the extent cooperative relationship-specific investments are important, this proposition points to an important advantage of the unilateral divorce rule.
3 The Marriage Decision and the Impact on Divorce Rates

3.1 Endogenous Marriage Model

To demonstrate that these results provide an explanation of the effect of the change in the divorce regime on divorce rates, I now consider a model where the marriage decision is endogenous. In this version of the model, a marriage occurs in period zero. In period one the two spouses observe their baseline utilities (utility before investment benefits) from the marriage and have the opportunity to divorce. If they do not divorce, then the investing spouse has the opportunity to make an investment. In period two, there is another opportunity to divorce and payoffs (utilities) are received. That is, spouses maximize expected period two utility. By allowing for divorce both in period one and in period two, I eliminate the problem of excessive investment that can occur under consent divorce (see Proposition 1). The non-investing spouse can compensate the investing spouse so that both are better off by divorcing in period one than having the investing spouse make an inefficient investment (because both spouses are completely informed in period one).

These periods occur as part of a seven date world with three marriages. (The periods represent time within a marriage, the dates represent absolute time.) Period zero for the first marriage occurs at date zero. Period zero for the second marriage occurs at date two (the last divorce date for the first marriage), and period zero for the third marriage occurs at date four (the last divorce date for the second marriage), so the final divorce decision occurs at date six. The consent regime governs in through date 2. Then in date 3 there is an unanticipated shift to the unilateral divorce regime. The following timeline describes the structure of the model.

<table>
<thead>
<tr>
<th>Date 0</th>
<th>Date 1</th>
<th>Date 2</th>
<th>Date 3</th>
<th>Date 4</th>
<th>Date 5</th>
<th>Date 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>marriage 1</td>
<td>invest or divorce 1</td>
<td>div. 1, marr. 2</td>
<td>invest or div. 2</td>
<td>div. 2, marr. 3</td>
<td>invest or div. 3</td>
<td>divorce 3</td>
</tr>
<tr>
<td>Consent</td>
<td>Consent</td>
<td>Consent</td>
<td>Unilateral</td>
<td>Unilateral</td>
<td>Unilateral</td>
<td>Unilateral</td>
</tr>
</tbody>
</table>

Let there be three possibility marriage utilities in period one for any marriage, $u_1 > 0 > u_2 > u_3$ with $u_1 + u_2 > 0$ and $u_1 + u_3 < 0$. Let the probability that either spouse has a period one utility of $u_j$ be $p_j$ with $p_1 + p_2 + p_3 = 1$. Say that a potential spouse can choose between two potential marriages, one where there will be an opportunity for one spouse to make a selfish investment (marriage $S$) and one where there will be an opportunity for one spouse to make a cooperative investment (marriage $C$). I assume that each spouse is equally likely to be the investing spouse in period zero. To simplify the exposition, assume that $b_I = b_N = b$. Furthermore, assume
that \(-(u_2 + u_3) > b > -2u_2, -(u_1 + u_3)\), that is, investing makes the marriage ex post efficient unless one spouse’s utility from the marriage is \(u_3\) and the other’s is either \(u_2\) or \(u_3\). The cost of making an investment of type \(k \in \{S, C\}\) is \(c_K\). I will assume that prior to time zero \(c_K\) is a random variable with cumulative distribution function \(F_K\) and probability density function \(f_K\) with support \([0, b]\) but that both spouses know the value of \(c_K\) in period zero (when deciding which marriage to choose).

Naturally, the spouse choosing between the two potential marriages will choose the one with the highest expected utility (net of expected investment costs). Of course, since the deciding spouse is equally likely to be the investing or the non-investing spouse (and no-investment marriage utilities are symmetric), she will choose the marriage that maximizes the total net expected utility of both spouses. Alternatively, the same decision rule results if one assumes that the deciding spouse knows whether or not she is the investing spouse but one allows for pre-marriage bargaining between the spouses with symmetric information. Thus, to determine the marriages that will result under each regime and the equilibrium divorce rate, one must first compare the total net expected utility from each type of marriage in each regime. After so doing, one can then determine which type of marriages will exist under which regime and calculate the expected divorce rate under each type of marriage and under each regime. The following proposition describes the results.

**Proposition 3 A.** Under consent divorce (marriage 1 and marriage 2), marriage \(S\) is always chosen over marriage \(C\). Under the unilateral divorce regime (marriage 3), either marriage \(C\) or marriage \(S\) could be chosen.

**B.** The probability of divorce is greater for marriage 2 (the first potential divorce under unilateral divorce) than for marriage 1 (under consent divorce). The probability of divorce for marriage 3 (the second potential divorce under unilateral divorce) is weakly smaller than it is for marriage 2 if (i) \(2u_2 \leq u_1 + u_3\) and \(b \leq -2u_2(1 + \frac{p_1 + p_2}{p_2 + p_3})\) or (ii) \(2u_2 > u_1 + u_3\) and \((b \leq -\frac{2u_2 p_2 + (u_1 + u_3) p_3}{p_2 + p_3} - \frac{2u_2 (p_1 + p_2)}{p_2 + p_3})\) or \(p_1 p_3 \geq p_2^2\). The probability of divorce in marriage 3 could be higher or lower than for marriage 1.

**Proof.** See Appendix.

This proposition demonstrates the effect of the shift from the consent divorce regime to the unilateral divorce regime. Under the consent regime, people always choose the marriage where there is an opportunity to make a selfish investment (as shorthand and no with pejorative implication intended, I will sometimes refer to this as a selfish marriage) rather than the marriage that presents
the opportunity to make a cooperative investment (which I will sometimes refer to as a cooperative marriage). No matter how much more productive the cooperative investment opportunity might be, since consent divorce provides no incentive to make such an investment, this opportunity has no value. When the divorce regime changes, the incentive to make selfish relationship-specific investments is reduced (relative to the consent regime), but now there is also some incentive to make cooperative relationship-specific investments. The incentive to make a cooperative investment is always less than the incentive to make a selfish investment for the same benefit and the same investment costs. But, if the cost of making the cooperative investment is sufficiently smaller than the cost of making the selfish investment, then the cooperative marriage can yield a higher expected utility than the selfish marriage.

The proposition also shows how the endogenous choice of marriage affects the divorce rate when there is an unanticipated shift in the divorce law. Marriage 2 has a higher divorce rate than marriage 1 since both marriages are chosen assuming that the consent regime is in place (so the selfish marriage is always chosen), but when the investment decision is made in marriage 2, the divorce regime has changed to the unilateral regime. Since the unilateral regime does not support selfish investments as strongly as the consent regime, the probability of divorce necessarily goes up. The difference between marriage 2 and marriage 3 is that while investments are made under the same divorce regime in both marriages, the choice of marriage type for the second marriage is made under incorrect expectations about what the divorce regime will be, while marriage 3 is chosen with correct expectations. This means that as long as the type of marriage that generates larger joint utility is also the type of marriage that has a lower probability of divorce, the divorce rate will be lower in marriage 3 than in marriage 2. The conditions in the proposition ensure that for any possible distribution for the investment costs for both cooperative and selfish investments that this is the case. That is, they guarantee that there is no combination of costs for the two types of investments such that a couple’s expected net utility is ever higher from choosing the cooperative marriage when the cooperative marriage also has a higher probability of divorce. These conditions are not necessary, since it could be that even if sometimes a couple chooses a marriage with a higher probability of divorce, on the average they will not do so.12 Thus, the proposition suggests that the initial shift to unilateral divorce should first increase divorce rates, but that then divorce rates

12The marriage with the higher probability of divorce could generate more utility if it generates much higher utility when there is no divorce. As the proof to the proposition shows, this is most likely when the selfish investment is cheap enough to be done quite frequently, so it saves the marriage frequently, but it is expensive enough that it doesn’t increase joint net utility over divorce by very much when it is undertaken.
should (under reasonable conditions) fall. The last part of the proposition says that the divorce rate could end up either above or below the divorce rate under consent divorce. The consent regimes supports one type of investment better than the unilateral regime supports either type of investment. But, the unilateral regime allows a couple to choose between two different types of marriage, so if the cooperative opportunity is more productive than the selfish opportunity with high enough probability, the probability of divorce in the unilateral regime could be lower than the probability of divorce in the consent regime.

3.2 Relation to Recent Empirical Findings

Becker (1981) and Becker et. al. (1977) argue that the introduction of unilateral divorce simply re-assigns the property rights between spouses, so the Coase theorem suggests that while this change will affect the distribution of rents between the spouses, it should not affect divorce rates. Coasian bargaining between spouses should ensure that divorce occurs if and only if it maximizes joint welfare under either the consent regime or the unilateral divorce regime. Recent empirical research on the effect of unilateral divorce laws on divorce, however, finds that the introduction of unilateral divorce did have significant, although temporary, impacts on divorce rates (Wolfers 2003). In particular, Wolfers finds that divorce rates rose sharply due to the adoption of unilateral divorce, but that within a decade unilateral divorce had no effect on the divorce rate. In fact, he finds that 15 years after the introduction of unilateral divorce, divorce rates are slightly lower than they would have been under the consent divorce regime.13

While this effect is puzzling under a theory of ex post Coasian bargaining, it is completely consistent with the results of my model of endogenous marriage choice with non-contractible marital investments. Under the consent divorce regime, Proposition 1B shows marriage works quite well at inducing spouses to make investments that increase their own marriage utility (sometimes, even a little too well). Proposition 2B shows, however, that the consent divorce regime works very poorly for inducing cooperative marital investments. Of course, both types of investments could be quite important in maintaining a relationship, but it is almost certainly true that in some relationships selfish investments are more important while in others cooperative relationships are likely to have bigger effects. Since marital matching is not exogenous, one should expect people to choose their

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13 Earlier empirical studies of the effect of unilateral divorce on divorce rates yielded conflicting results (Friedberg 1998, Peters 1986 and 1992, Allen 1992). Wolfers’ advance over the previous studies is that he explicitly takes into account the dynamic nature of the response to the change in the policy regime.
spouses with an eye to what types of investments are likely to be more important in their marriage.

If the legal rule is consent divorce, it makes more sense to get into a marriage where selfish investments are likely to be important and cooperative investments less so. When the legal rule changed to unilateral divorce, people who got into relationships where selfish investments were likely to be important found themselves in a legal regime that provided insufficient incentives for these investments (Proposition 1A). So, it should come as no surprise that divorce rates increased (as Proposition 3 predicts). But, the people contemplating marriage after the introduction of unilateral divorce (or even shortly before it for those who saw the writing on the wall) should have started to choose their spouses differently than those who married before the change in legal regime. Now, choosing relationships where cooperative investments could be very productive is not pointless. If there are enough potential marriages where cooperative investments can make a big impact on marital utility at reasonable costs, then the unilateral divorce regime may cause many spouses to choose these marriages rather than alternative marriages where the benefits from selfish investments are more modest. Once these more recent marriages start to make up a large percentage of total marriages, Proposition 3 shows that one should expect the divorce rate to fall.

Not only is this story entirely consistent with Wolfers’ findings regarding the divorce rate, it is also exactly what Mechoulan (2005) finds when comparing the effect of the change in the divorce regime on couples who married under the consent regime versus those who married under the unilateral regime. His analysis shows that the change in the divorce regime significantly increased the probability of divorce for those couples who married under the old regime, it had no significant effect on the divorce probability for those couples who married under the new regime. In my model, whether the unilateral divorce rate ends up above or below the divorce rate under the consent regime should depend on whether selfish or cooperative investments are more important in the average marriage. The fact that divorce rates may have settled slightly below what they were prior to the introduction of unilateral divorce suggests that maybe cooperative investments are more important.

Further evidence for the importance of cooperative investments, when considered in the context of my model of non-contractible marital investments, can be found in another recent empirical paper about the effects of unilateral divorce. Stevenson and Wolfers (2003) find that the introduction of unilateral divorce led to significant declines in domestic violence and female suicides.\footnote{Brinig and Crafton (1994) get the opposite result. The Wolfers and Stevenson study, however, measures domestic violence more accurately and has data on domestic violence from two years, allowing them to use difference in}
domestic violence is an action that one spouse takes that harms the other, an investment in making oneself less prone to commit domestic violence (going to therapy, for example) is largely (if not entirely) a cooperative investment. Of course, suicide is something one does to oneself. Still, it is hard to imagine that anyone would not have the incentive to make a selfish investment that would prevent their own suicide. A more plausible story is that married women commit suicide when external factors make their life unbearable. One of these external factors could be behavior of their husbands. It is easy to imagine that there are many investments a husband could make that might significantly reduce the probability of his wife committing suicide (stopping domestic violence is an obvious example).

Of course, the Coasian bargaining framework in my model suggests marriages where there is spousal abuse or where one spouse is led to commit suicide should lead to divorce before these effects are realized. Surely, such marriages have negative joint welfare. This framework, however, assumes completely transferable utility. In marital bargaining, however, one can easily imagine that utility is not completely transferable.\footnote{Clark (1999) and Fella et. al. (2004) have models of the effect of divorce laws with non-transferrable utility.} Allowing for this does not undermine the result that unilateral divorce provides greater incentives for cooperative investment, but it does explain why we might not see divorces under the consent regime where there is domestic violence. Once the regime switches to unilateral divorce, however, the greater incentives for cooperative investment should induce more spouses to make investments that make them less prone to domestic violence.

That said, one might also argue that if non-transferable utility is important, the reason unilateral divorce leads to a reduction in domestic violence is that it leads to more divorces rather than because it leads to more cooperative investment. The fact that the steady state divorce rate under unilateral divorce appears to be as low or lower as it would have been under consent divorce, however, suggests that a large part of the effect of unilateral divorce must be through some other channel. This is especially true given that, ignoring investment incentives, one's incentive to avoid a marriage where domestic violence is likely is greater under consent divorce than unilateral divorce if non-transferrable utility impedes bargaining. Given that there is strong empirical evidence that spousal behavior does respond to the incentives created by changes in divorce laws (see Chiaporri et. al. (2002) showing changes in divorce laws affect female labor supply in marriage), it is reasonable to assume that this channel is the effect these laws have on behavior during marriage. The model in this paper shows that one important effect of unilateral divorce is the increased incentive it
provides for cooperative investments.

4 Unilateral Divorce v. No Marriage

The change from a consent divorce regime to a unilateral divorce regime has greatly reduced the
difference between married relationships and unmarried relationships since, in both cases, either
partner can terminate the relationship on her own. In fact, with this change, one might wonder
if there is now, under unilateral divorce, any difference in the investment incentives facing married
versus unmarried couples. This section argues that there is an important feature of marriage, the
cost of getting divorced, that does have an important, and potentially socially beneficial, effect
on relationship-specific investment incentives. In essence, this is another example of a situation
where parties might want to commit to an ex post inefficient outcome to improve their incentives
for ex ante investment. In particular, this section will show that making divorce costly (but not
too costly) for both partners can change the equilibrium in a two-sided investment game from no
investment by both spouses to investment by both spouses. Furthermore, there are situations
where doing so increases the ex ante joint surplus from the relationship. Of course, there are also
situations, where divorce is efficient, where costly breakup reduces joint surplus. If this effect is
likely to dominate the incentive effects of costly divorce, however, the couple can always choose not
to marry.

4.1 Selfish Relationship-Specific Investments

Consider the case where \( u_A \leq u_B < 0 \) (so that both spouses want a divorce if neither invests), \(^{16}\) and
where both \( A \) and \( B \) have the opportunity to make an investment that increases their own marriage
utility by \( b_A \) and \( b_B \) respectively. These investments cost the spouses \( c_A \) and \( c_B \) respectively. If
for one spouse, call this spouse \( A \), marriage utility after this investment is less than the cost of the
investment, \( u_A + b_A - c_A < 0 \), then he will not make this investment even if her partner invests if
divorce is costless. Spouse \( B \)'s payoff from investing when \( A \) does not is (with costless divorce)
\( u_A + u_B + b_B - c_B \) if doing so saves the marriage (since she has to compensate \( A \) for staying in the
marriage), her payoff is \(-c_B\) otherwise. So, if \( u_A + u_B + b_B - c_B < 0 \), the unique equilibrium when
divorce is costless is for neither spouse to invest. This gives each spouse a joint payoff of zero.

\(^{16}\)Obviously, this would not represent a situation that occurs immediately after marriage. Since marriage utility
is subject to shocks, however, many marriages should find themselves in this situation at some point.
Notice, however, that as long as \( b_A - c_A \in (-(u_A + u_B + b_B - c_B), -u_A) \), the combined net payoff of the couple is positive if both invest. The Nash equilibrium with costless divorce is inefficient. Making divorce costly can solve this problem. Say the cost of divorce is \( d > 0 \). (Each spouse’s divorce utility is \( -d < 0 \)). If \( d \) is large enough that \( u_A + d + u_B + b_B - c_B > -d \), then \( B \)'s dominant strategy is to invest. To see this, note that if \( B \) invests and \( A \) does not, then \( B \) must compensate \( A \) by \( -(u_A + d) \) to induce \( A \) to stay married. This gives \( B \) a marriage utility of \( u_A + d + u_B + b_B - c_B \). If \( d \) is large enough, this exceeds her payoff from divorce, \( -d \). So, her best response to \( A \) not investing is to invest herself. If \( A \) does invest, then \( B \)'s payoff from investing is even higher (she has to compensate \( A \) less, if at all) and her payoff from not investing remains at \( -d \) (either because there is a divorce or because \( A \) compensates \( B \) so that \( B \) gets her divorce utility). To induce \( A \) to invest if \( B \) does, it must be that \( u_A + b_A - c_A > -d \) (her marriage utility exceeds her divorce utility). So, any \( d > \max\{-(u_A + u_B + b_B - c_B), -(u_A + b_A - c_A)\} \) will change the unique Nash equilibrium from no investment by both spouses to investment by both spouses.

For \( b_A - c_A \in (-(u_A + u_B + b_B - c_B), -u_A) \), this change to costly divorce increases net joint surplus. Of course, spouse \( A \) would be better off if divorce were costless since her ex post payoff is negative. But, at the time the couple is deciding whether or not to get married, they may not know which spouse will be spouse \( A \). That is, they may expect that at some point in the marriage both spouses will have to put in some effort to make the marriage work, and that even after putting in this effort, one spouse may benefit from the marriage a lot more than the other. But, since this event may be due to some random shocks, each may feel they have a reasonable chance of being in the position of the spouse that benefits more. So, both may be better off in expectation by getting married and thereby making it more costly to breakup.

### 4.2 Cooperative Relationship-Specific Investments

Consider the case where \( u_A \leq u_B < 0 \), and where both \( A \) and \( B \) have the opportunity to make an investment that increases the marriage utility of their spouse by \( b_B \) and \( b_A \) respectively. Again, these investments cost the spouses \( c_A \) and \( c_B \) respectively. Further, suppose that \( u_B + b_B > 0 \) while \( u_A + b_A < 0 \). By Proposition 2, if divorce is costless, we know that neither spouse will make the cooperative investment if the other spouse does not, since if spouse \( j \) does not make the cooperative investment, then spouse \(-j\) does not have a positive marriage utility. So, if divorce is costless, there is a Nash equilibrium where neither spouse invests. In fact, since \( u_A + b_A < 0 \),
spouse $A$ will not make this cooperative investment regardless of what $B$ does. The unique Nash equilibrium has neither spouse investing, resulting in divorce.

Making divorce costly, however, can generate a Nash equilibrium where both spouses invest that generates positive joint surplus for the couple. The following example will illustrate how this can happen. Let $u_A = u_B = -6$, but say that both spouses have the opportunity to make a cooperative investment that raises the other spouse’s marriage utility. The cost of such an investment is the same for both spouses ($c_A = c_B = 1$), but $A$’s investment increases $B$’s marriage utility much more than the reverse ($b_B > b_A = 5$). The net joint surplus if both spouses invest is $u_A + b_A + u_B + b_B - c_A - c_B = b_B - 9$. So, for $b_B > 9$, inducing both spouses to invest increases net joint surplus over the costless divorce equilibrium.

To determine under what circumstances costly divorce supports an equilibrium where both spouses invest, consider each spouse’s best response to investment by the other spouse. First, consider $A$’s best response to investment by $B$. If $A$ assumes that if she does not invest, there will be a divorce ($u_A + b_A + u_B < -2d$ or $d < 7/2$), then she prefers to invest and save the marriage if and only if $u_A + b_A - c_A > -d$ or $d > 2$. On the other hand, if $A$ assumes that if she does not invest, the marriage will survive but she’ll have to compensate $B$ to prevent the divorce, the marriage is in state $B$ Tolerates $A$ ($u_A + b_A + u_B > -2d$ or $d > 7/2$), then her payoff if she does not invest is $u_A + b_A + u_B + d$. This means she prefers to invest if and only if $u_A + b_A - c_A > u_A + b_A + u_B + d$. This simplifies to $-c_A > u_B + d$ or $d < 5$. So, if $d \in (2, 5)$, $A$’s best response to $B$’s investment is to invest also.

Now, consider $B$’s best response to investment by $A$. If $B$ assumes that if she does not invest, there will be a divorce ($u_A + u_B + b_B < -2d$ or $d < (12 - b_B)/2$), then she prefers to invest and save the marriage if and only if $u_B + b_B - c_B > -d$ or $d > 7 - b_B$. Of course, for $b_B > 9$, this is inconsistent with making investment $A$’s best response to $B$’s investment. For $b_B > 9$, the relevant case is where $B$ assumes that if she does not invest, the marriage will survive but she’ll have to compensate $A$ to prevent the divorce. The marriage is in state $A$ Tolerates $B$ ($u_A + u_B + b_B > -2d$ or $d > (12 - b_B)/2$), so her payoff if she does not invest is $u_B + b_B + u_A + d$. This means she prefers to invest if and only if $u_B + b_B - c_B > u_B + b_B + u_A + d$. This simplifies to $-c_B > u_A + d$ or $d < 5$. So, if $b_B > 9$ and $d \in (2, 5)$, $B$’s best response to $A$’s investment is to invest also. Thus, by making divorce costly, but not too costly, one can support a Nash equilibrium where both spouses invest.

In this Nash equilibrium, the couple stays married and joint surplus is positive. Granted, there is still another Nash equilibrium where neither spouse invests and the couple gets divorced. But,
if divorce is costly, both spouses are better off with the investment Nash equilibrium, so one would expect they could coordinate on this equilibrium. As was true in with cooperative investments, one spouse, spouse $A$, would be better off still if divorce were costless since her ex post payoff is negative. But, as discussed above, if the couple does not know which spouse will be spouse $A$, both may prefer using marriage to make breakup costly.

5 Conclusion

The shift from consent to unilateral divorce represented a major policy change on an issue that affects a huge fraction of the population. While there have been many empirical papers attempting to estimate the effect of this change on divorce rates and other measures of well-being, there has been no rigorous theoretical analysis of its effects on the incentives of married people to invest in their relationships. This paper fills that gap. In so doing, it provides detailed results on the advantages and disadvantages of each divorce regime. Not only does it provide a framework that helps to explain the empirical results on the effects of the move to unilateral divorce, but it also suggests other effects of the change that may be harder to measure but also quite important. For example, not only does this model predict (as Stevenson and Wolfers (2003) have shown empirically) that unilateral divorce should lead to a reduction in spousal abuse, it also predicts that unilateral divorce should lead to an increase in cooperative relationship-specific investments of all types. While selfish relationship-specific investments are undoubtedly important, it is not obvious which regime provides better incentive for these investments. Incentives are stronger under the consent regime, but they are often too strong, whereas under unilateral divorce, these incentives are often too weak. But, for cooperative investments, unilateral divorce unambiguously provides incentives that are closer to the socially optimal incentives. This, combined with empirical evidence suggesting the importance, at least for the most extreme cases, of this type of investment, presents a strong argument in favor of unilateral divorce. That said, the fact that the consent divorce regime is superior for some types of marriage suggests that the best rule might be one that gives a married couple the option to choose which divorce regime would govern their divorce. With such an option, a couple that believes selfish investments will likely play a bigger role in their marriage could choose the consent regime, while a couple that believes cooperative investments are likely to be more important could choose the unilateral regime. The only downside to such a system would be if the choice of regime were sometimes used to signal some unobservable characteristic. In that case, a couple might not
choose the divorce regime that maximized their expected joint welfare.

The results in this paper also caution against making divorce too easy (or too hard). Making a breakup costly can sometimes be essential to moving the couple to a more efficient equilibrium where both spouses make investments in the marriage. Because it is very difficult, if not impossible, for a couple to commit to making their breakup (ex post) inefficiently costly, the law can play a valuable role in enabling this commitment. Of course, costly breakup has the disadvantage of wasting resources when shocks make separation efficient. But, since marriage is voluntary, costly divorce gives couples the option of choosing the level of commitment that maximizes their joint welfare.

6 Appendix

Proof of Proposition 3. For this proof, define \( \pi_K = \int_0^b cf_K(c)dc \) and \( \pi_K(x) = \int_0^x cf_K(c)dc / F_K(x) \). That is \( \pi_K \) is the mean cost of an investment of type \( K \), and \( \pi_K(x) \) is the mean cost of an investment of type \( K \) conditional on the cost being less than \( x \).

A. First, consider the consent regime. In marriage \( C \), then by Proposition 2, we know that \( I \) will not make an investment. Thus, the marriage will end in divorce with probability:

\[
p_3 + (1 - p_3)p_3 + p_2^2
\]  \hspace{1cm} (1)

Total net expected utility (of both spouses) from marriage \( C \) in the consent regime is:

\[
p_1^2(2u_1) + 2p_1p_2(u_1 + u_2)
\]  \hspace{1cm} (2)

In marriage \( S \), however, by Proposition 1 we know that \( I \) will invest (with probability 1) in the marriage if the marriage would survive even without the investment (this happens with probability \( 1 - [p_3 + (1 - p_3)p_3 + p_2^2] \)). If there will be divorce even with the investment, then \( I \) could increase her divorce payoff by \( b/2 \) by investing. But, since investing is inefficient, \( N \) will compensate \( I \) \( b/2 - c_S \) more to obtain a divorce in period one rather than in period 2.\(^{17}\) So, \( I \) will not invest. If both spouses would have utility \( u_2 \) without the investment, then by investing \( I \) increases her payoff by \( b + u_2 \), but it only increases joint payoffs by \( 2u_2 + b \) (recall that \( u_2 < 0 \)). So, if \( c_S \in (2u_2 + b, u_2 + b) \),

\(^{17}\)While not critical to the result, \( N \) need only pay \( I \) \( b/2 - c_S \) because \( I \)'s next best alternative, investing, is an outside option.
then, again, \( N \) will pay \( I \) to obtain a divorce in period one. So, the probability that \( I \) will invest is \( F_S(b + 2u_2) \).\(^{18}\) If without the investment one spouse’s utility would have been \( u_1 \) and the other \( u_3 \), then investing increases \( I \)’s utility by \( b + (u_1 + u_3)/2 \), but it only increases joint utility by \( b + (u_1 + u_3) \). So, the probability that \( I \) will invest is \( F_S(b + u_1 + u_3) \). If there is investment in either of these cases, then investment saves the marriage. So, the total probability of divorce is:

\[
p_3(p_3 + 2p_2) + 2(1 - F_S(b + u_1 + u_3))p_1p_3 + (1 - F_S(b + 2u_2))p_2^3
\]

Total net expected utility (of both spouses) from marriage \( S \) (prior to period zero, when \( c_S \) is unknown) in the consent regime is:

\[
p_1^2(2u_1 + b - \tau_S) + 2p_1p_2(u_1 + u_2 + b - \tau_S) + p_2^2F_S(b + 2u_2)(2u_2 + b - \tau_S(b + 2u_2)) + 2p_1p_3F_S(b + u_1 + u_3)(u_1 + u_3 + b - \tau_S(b + u_1 + u_3))
\]

Notice, however, that (4) represents the expected net utility prior to period zero, and not the expected utility from the marriage at period zero, when \( c_S \) is known. To determine which marriage is chosen, I need to find expected utility from the marriage as a function of \( c_S \). If we define \( \delta_S(z) \) to be the indicator function that equals 1 if \( c_S < z \) and 0 otherwise, then the total net expected utility (of both spouses) from marriage \( C \) at time zero in the consent regime is:

\[
p_1^2(2u_1 + b - c_S) + 2p_1p_2(u_1 + u_2 + b - c_S) + p_2^2\delta_S(b + 2u_2)(2u_2 + b - c_S) + 2p_1p_3\delta_S(b + u_1 + u_3)(u_1 + u_3 + b - c_S)
\]

Now, it is easy to see that (2) is less than (5) for any \( c_S < b \).\(^{19}\) That is, marriage \( S \) will be chosen over marriage \( C \).

Now consider the unilateral regime. In marriage \( C \), Proposition 2 tells us that \( I \) will not invest unless her marriage utility is \( u_1 \) and \( N \)’s marriage utility is \( u_2 \) or \( u_3 \). If the no investment marriage utilities are \( u_1, u_2 \) then investing increases \( I \)’s payoff by \(-u_2 \) (she no longer has to pay \((-u_2) \) to stop a divorce). So, \( I \) will invest if and only if \( c_C < -u_2 \). If the no investment marriage utilities

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\(^{18}\)Of course, at the time the marriage is chosen, the spouses know \( c_S \), so the investment decision is deterministic. But, from the point of view of predicting the expected divorce probability (or the expected utility from this marriage), one can treat \( c_S \) as an unknown random variable.

\(^{19}\)This option to divorce in period one of a marriage is critical here because it eliminates the possibility that there will be inefficient selfish investments in marriage \( S \). Without that, it could be optimal to choose marriage \( C \) simply to foreclose the possibility of investment.
are \( u_1, u_3 \) then investing increases \( I \)'s payoff from zero to \( \text{Min}[u_1 + u_3 + b, u_1] \). If we define \( \delta_C(z) \) to be the indicator function that equals 1 if \( c_C < z \) and 0 otherwise, then the total net expected utility (of both spouses) from marriage \( C \) in the unilateral regime is:

\[
p_1^2(2u_1) + p_1p_2\{2(u_1 + u_2) + \delta_C(-u_2)(b - c_C)\} + p_1p_3\delta_C(\text{Min}[u_1 + u_3 + b, u_1])(u_1 + u_3 + b - c_C))
\]

(6)

While calculating the probability of divorce for marriage \( C \) in the unilateral regime is complicated by the selection effect (marriage \( C \) is more likely to be chosen when \( c_C \) is lower), it is useful to determine this probability ignoring this effect. If one ignores the selection effect, then \( I \) invests with probability \( F_C(-u_2) \) when marriage utilities are \( u_1, u_2 \) and with probability \( F_C(\text{Min}[u_1 + u_3 + b, u_1]) \) when marriage utilities are \( u_1, u_3 \). So, ignoring the selection effect, the probability of divorce is:

\[
p_3(p_3 + 2p_2) + (2 - F_C(\text{Min}[u_1 + u_3 + b, u_1]))p_1p_3 + p_2^2
\]

(7)

Alternatively, it is also useful to have the divorce probability in marriage \( C \) in the unilateral divorce regime given \( c_C \). This is:

\[
p_3(p_3 + 2p_2) + (2 - \delta_C(\text{Min}[u_1 + u_3 + b, u_1]))p_1p_3 + p_2^2
\]

(8)

In marriage \( S \), Proposition 1 says that \( I \) will invest with probability one if her marriage utility is \( u_1 \) and \( N \)'s is \( u_1 \) or \( u_2 \). If \( I \)'s marriage utility is \( u_1 \) and \( N \)'s is \( u_3 \), then investing increases \( I \)'s payoff by \( b + u_1 + u_3 \). If \( I \)'s marriage utility is \( u_2 \) (respectively, \( u_3 \)) and \( N \)'s is \( u_1 \), then investing increases \( I \)'s payoff by \( b + u_2 \) (respectively, \( \text{Max}[b + u_3, 0] \)). If both spouses have a marriage utility of \( u_2 \), then investing increases \( I \)'s payoff by \( b + 2u_2 \). Otherwise, \( I \) will not invest. Total net expected utility (of both spouses) from marriage \( S \) in the unilateral regime is:

\[
p_1^2(2u_1 + b - c_S) + p_1p_2\{2(u_1 + u_2) + b - c_S + \delta_S(b + u_2)(b - c_S)\} + p_2^2\delta_S(b + u_2)(2u_2 + b - c_S) + p_1p_3[\delta_S(b + u_1 + u_3) + \delta_S(b + u_3)](u_1 + u_3 + b - c_S)
\]

(9)

Ignoring the selection effect, one can analyze the probability of divorce as follows. If marriage utilities are \( u_1, u_3 \), then \( I \) will invest with probability \( F_S(b + u_1 + u_3) \). If marriage utilities are \( u_2, u_1 \) (respectively, \( u_3, u_1 \)), then \( I \) will invest with probability \( F_S(b + u_2) \) (respectively, \( F_S(b + u_3) \)). If
marriage utilities are \( u_2, u_2 \), then \( I \) invests with probability \( F_S(b+2u_2) \). This makes the probability of divorce (ignoring selection):

\[
p_3(p_3 + 2p_2) + (2 - F_S(b + u_1 + u_3) - F_S(b + u_3))p_1p_3 + (1 - F_S(b + 2u_2))p_2^2 \quad (10)
\]

The divorce probability in marriage \( S \) in the unilateral divorce regime given \( c_S \) is:

\[
p_3(p_3 + 2p_2) + (2 - \delta_S(b + u_1 + u_3) - \delta_S(b + u_3))p_1p_3 + (1 - \delta_S(b + 2u_2))p_2^2 \quad (11)
\]

Comparing (6) and (9), it is easy to see that if \( c_S \leq c_C \), then marriage \( S \) always provides more utility. When \( c_S \) is much larger than \( c_C \), however, then marriage \( C \) can provide more joint expected utility. To see this, imagine that \( c_S = b \). Then (9) becomes \( 2p_1^2u_1 + 2p_1p_2(u_1 + u_2) \), which is smaller than (6) so long as \( c_C < \text{Max}\{-u_2, \text{Min}[u_1 + u_3 + b, u_1]\} \).

B. Marriage 1 is a \( S \) marriage under consent divorce. So, the probability of divorce is given by (3). Marriage 2 is a \( S \) marriage under unilateral divorce (it is an \( S \) marriage since the marriage was chosen under the belief that the law was consent divorce). So, the true probability of divorce for marriage 2 is given by (10) (we do not have to worry about the selection effect since the marriage was chosen under the belief that the law was consent divorce, for which marriage \( S \) is preferred for any level of investment cost). Subtracting (3) from (10) gives \( (F_S(b+u_1+u_3) - F_S(b+u_3))p_1p_3 > 0 \).

Marriage 3 is a \( S \) marriage if and only if (9) \( \geq (6) \). If marriage \( S \) is chosen, then the probability of divorce is weakly less than (10 because of the selection effect. If marriage \( C \) is chosen, then (9) \( < (6) \), which implies:

\[
-p_1^2(b - c_S) + p_1p_2\{\delta_C(-u_2)(b - c_C) - [b - c_S + \delta_S(b + u_2)(b - c_S)]\} - p_2^2\delta_S(b + 2u_2)(2u_2 + b - c_S)\]
\[
+p_1p_3\{\delta_C(\text{Min}[u_1 + u_3 + b, u_1])(u_1 + u_3 + b - c_C) - [\delta_S(b + u_1 + u_3) + \delta_S(b + u_3)(u_1 + u_3 + b - c_S)]\}
\]

The probability of divorce can only increase if marriage \( C \) is chosen if there exists some \( c_C \) and \( c_S \) such that both (12) is positive and difference in probability of divorce conditional on the investments costs is also positive. The latter is given by (8) - (11), which is:

\[
(\delta_S(b + u_1 + u_3) + \delta_S(b + u_3) - \delta_C(\text{Min}[u_1 + u_3 + b, u_1]))p_1p_3 + \delta_S(b + 2u_2)p_2^2 \quad (13)
\]

Notice that (12) is increasing in \( c_S \) and decreasing in \( c_C \). Thus, for any given value of (13), (12)
is largest for the smallest \( c_C \) and the largest \( c_S \) that produce this value of (13). If \( \delta_C(\text{Min}[u_1 + u_3 + b, u_1]) = 0 \), so that (13) is necessarily non-negative, then (12) is decreasing in \( p_3 \). So, if it is negative at \( p_3 = 0 \), it is negative everywhere. But, if \( p_3 = 0 \), then we can maximize (12) without changing the value of (13) by setting \( c_C = 0 \). Thus, if one can show that at \( c_C = 0 \) that it is impossible for both (12) and (13) to be positive, then it is impossible for them to both be positive for any \( c_C \). If \( c_C = 0 \), then there are three possible values of (13), and it is sufficient to examine the largest possible values of \( c_S \) consistent with those values. That is, it suffices to show that it is impossible for both (12) and (13) to be positive at \( c_S = b + u_3 \), \( c_S = b + 2u_2 \), and \( c_S = b + u_1 + u_3 \).

First, I evaluate (12) at \( c_S = b + u_3 \) and \( c_C = 0 \) and show that it is negative:

\[
p_1^2(u_3) + p_1p_2\{b + 2u_3\} - p_2^2\delta_S(b + 2u_2)(2u_2 + b - c_S) + p_1p_3\{u_3 + b - 2u_1\} < 0 \tag{14}
\]

Since (12) is decreasing in \( c_S \), this also implies that (12) is negative at \( c_S = b + u_1 + u_3 \) and \( c_C = 0 \).

Next, consider \( c_S = b + 2u_2 \) and \( c_C = 0 \). (12) is clearly negative if \( 2u_2 \leq u_3 \) (because it is decreasing in \( c_S \)) so assume \( 2u_2 > u_3 \), which means that \( \delta_S(b + u_3) = 0 \). This makes (12):

\[
p_1^2(2u_2) + p_1p_2\{b + 4u_2\} + p_1p_3\{u_1 + u_3 + b - \delta_S(b + u_1 + u_3)(u_1 + u_3 - 2u_2)\} \tag{15}
\]

We consider two possible case, case 1: \( 2u_2 \leq u_1 + u_3 \) which implies that \( \delta_S(b + u_1 + u_3) = 1 \). Thus, (13) is positive and (12) becomes:

\[
p_1\{p_1(2u_2) + p_2(b + 4u_2) + p_3(b + 2u_2)\}
\]

This is negative if and only if:

\[
b \leq -2u_2(1 + \frac{p_1 + p_2}{p_2 + p_3})
\]

Case 2: \( 2u_2 > u_1 + u_3 \) which implies that \( \delta_S(b + u_1 + u_3) = 0 \), so this becomes:

\[
p_1\{p_1(2u_2) + p_2(b + 4u_2) + p_3(u_1 + u_3 + b)\} \tag{16}
\]

So, this is non-positive if

\[
b \leq -\frac{2u_2p_2 + (u_1 + u_3)p_3}{p_2 + p_3} - \frac{2u_2(p_1 + p_2)}{p_2 + p_3}
\]
So, either I need \( b \leq \frac{2u_2p_2 + (u_1 + u_3)p_3}{p_2 + p_3} \) (so that marriage \( S \) is chosen) or I need \( p_1p_3 \geq p_2^2 \), so that the divorce rate not higher with marriage \( C \).

Comparing the probability of divorce between marriage 1 and marriage 3, it is easy to see that either could be larger. The divorce rate in marriage 3 will be larger if \( c_C \) and \( c_S \) are perfectly correlated and equal to each other. In that case, marriage 3 will always be an \( S \) marriage, and the probability of divorce in marriage 3 will be equal to the probability of divorce in marriage 2, which is greater than the probability of divorce in marriage 1. At the other extreme, if \( c_C = 0 \) is probability one and \( c_S = b \) with probability one, then marriage 3 will always be a \( C \) marriage and the probability of divorce in marriage 1 will be \( p_3(p_3^2 + 2p_2 + p_1) + p_2^2 \) and the probability of divorce in marriage 3 will be \( p_3(p_3^2 + 2p_2 + p_1) + p_2^2 \), which is clearly smaller. Q.E.D.
References


